The unshocked ejecta in Cas A and Tycho through low-frequency radio absorption

María Arias, Jacco Vink, Francesco de Gasperin, +

Chania, June 2019

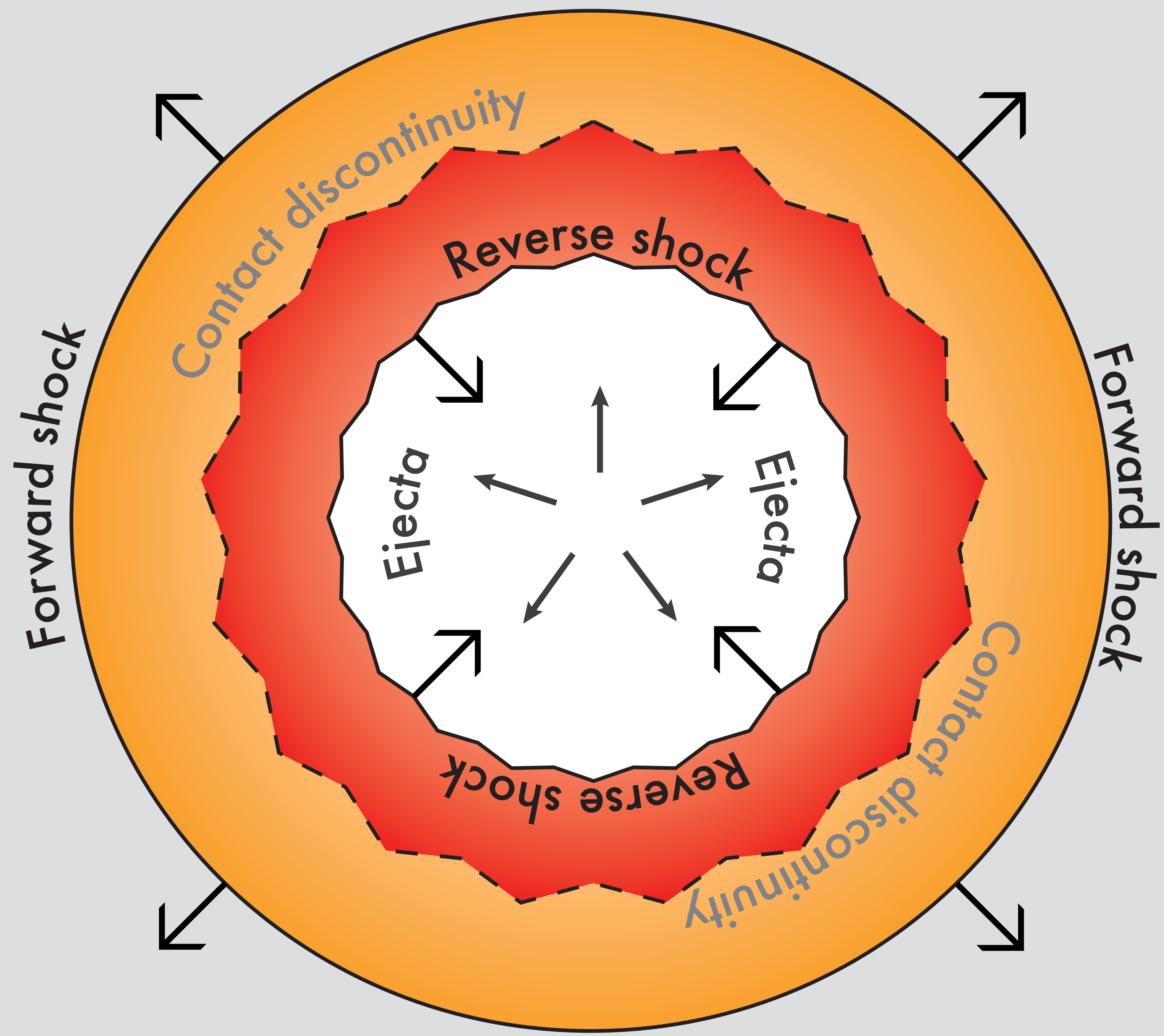
ANTON PANNEKOEK INSTITUTE

The LOw Frequency ARray

- Interferometer with stations in the Netherlands (24 *Core*, 14 *Remote*) and across Europe (12 *International*)

- 30-90 MHz (**LBA**);
110-240 MHz (**HBA**)

LOFAR Superterp, which houses six stations.



Vink 2020, in prep.

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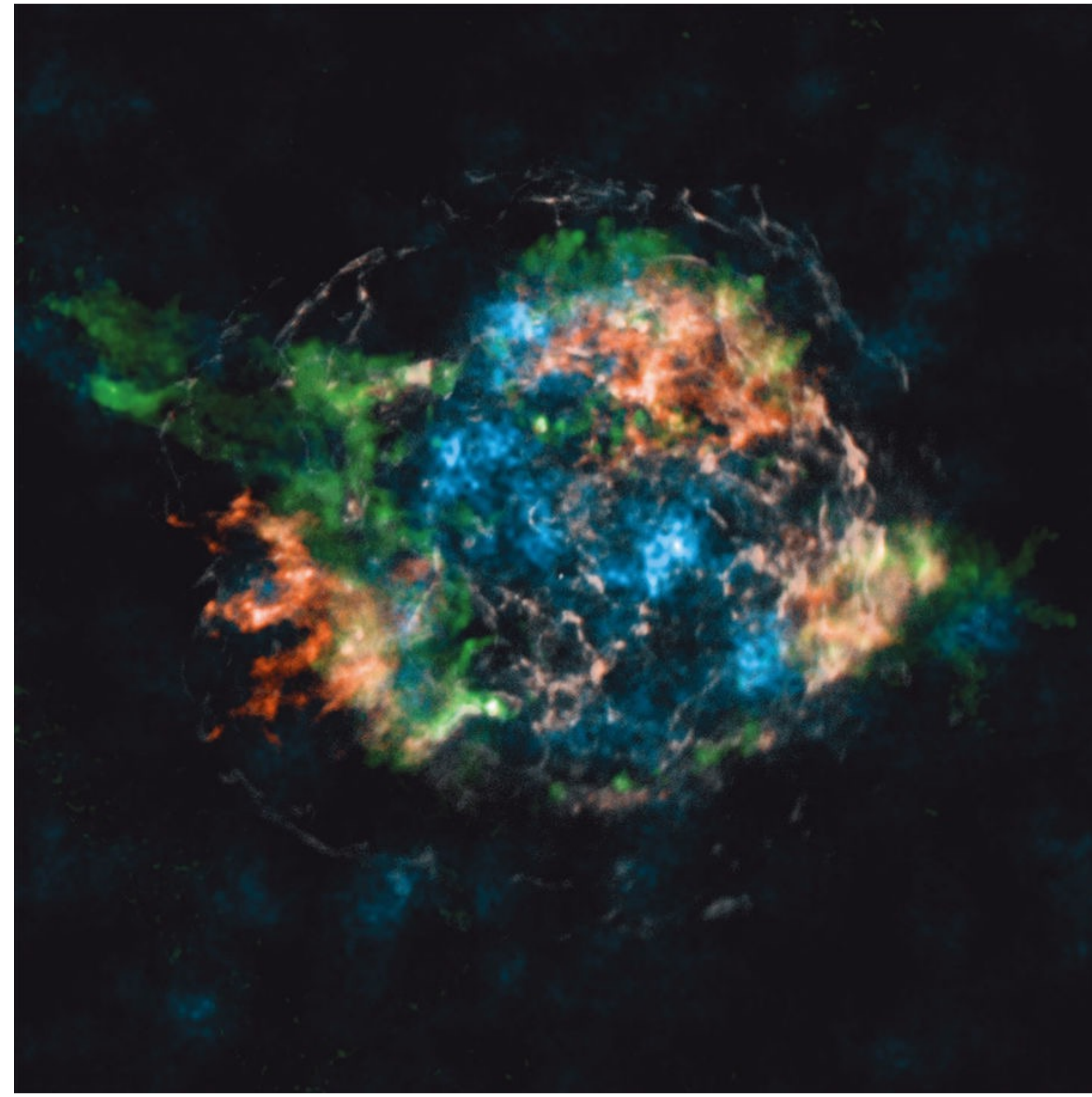
- Radioactive decay of elements synthesised in the explosion (^{44}Ti has $t_{1/2}$=60 years) [Grefenstette+ 16]



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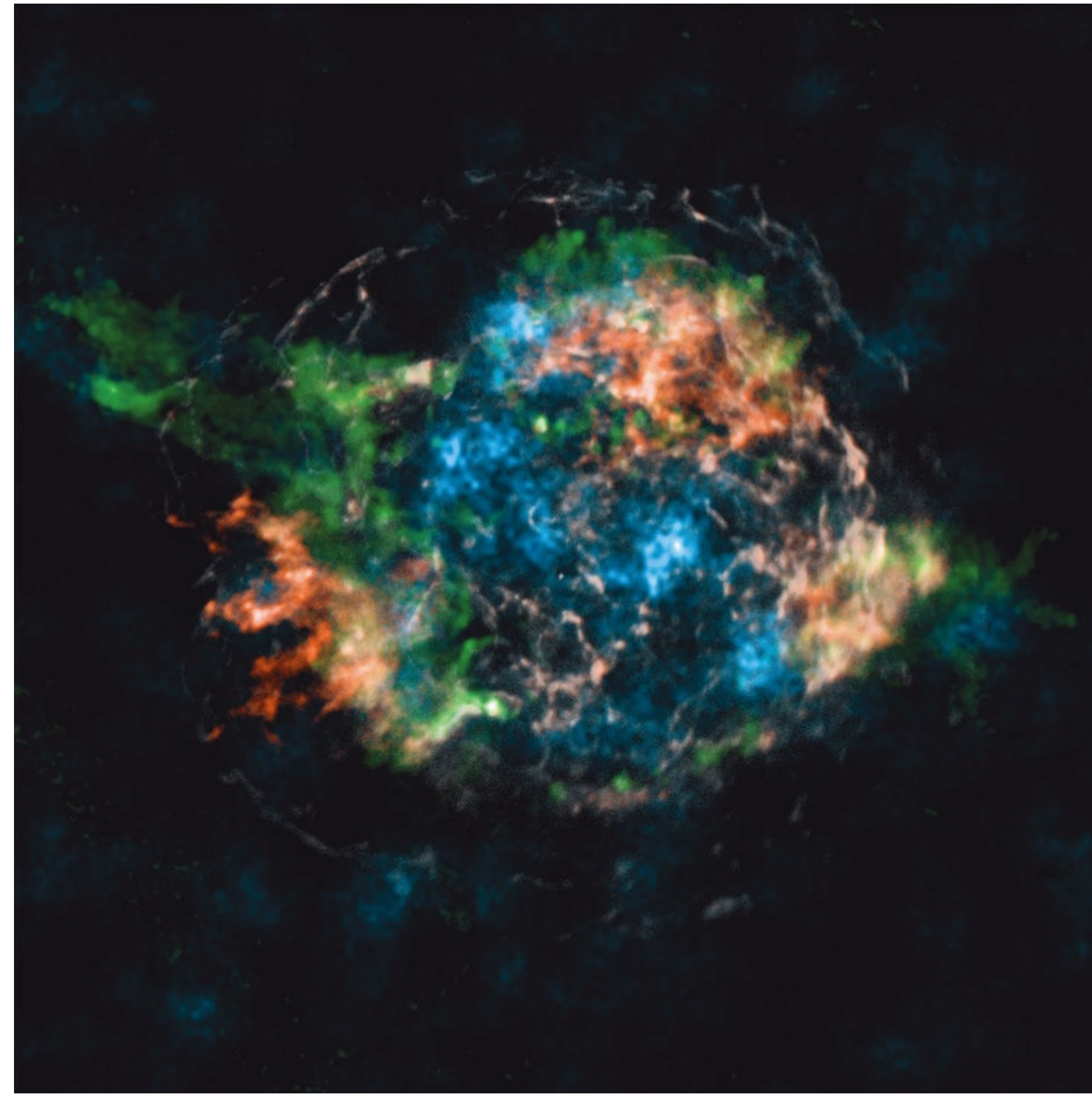
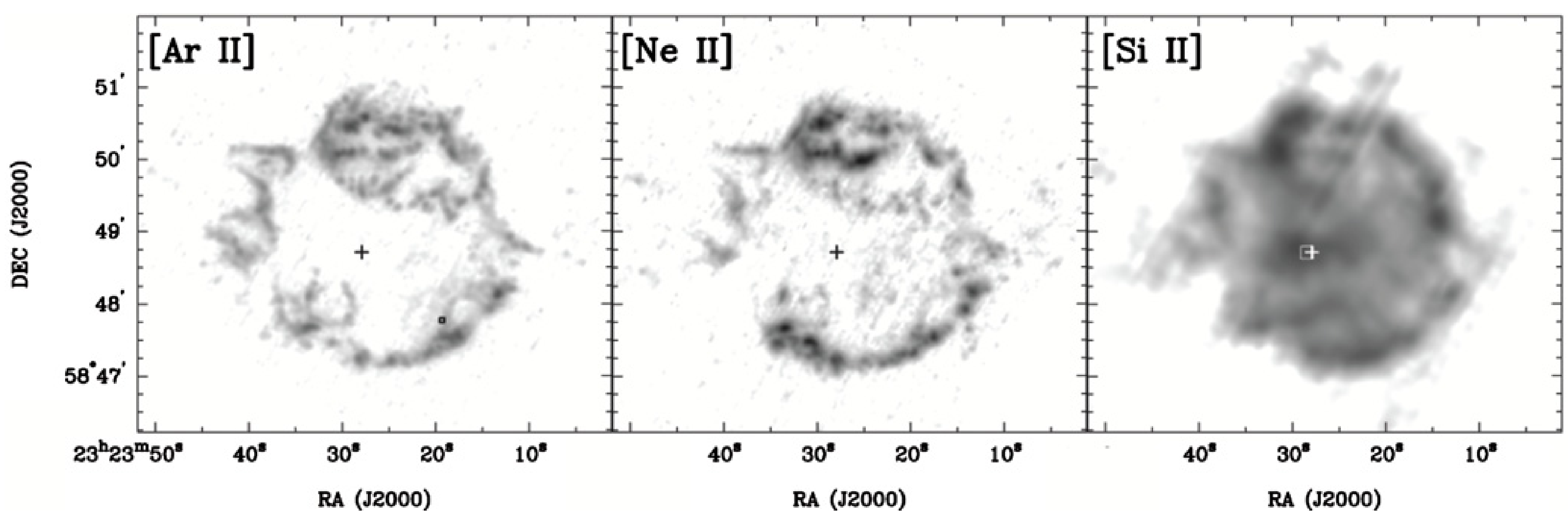
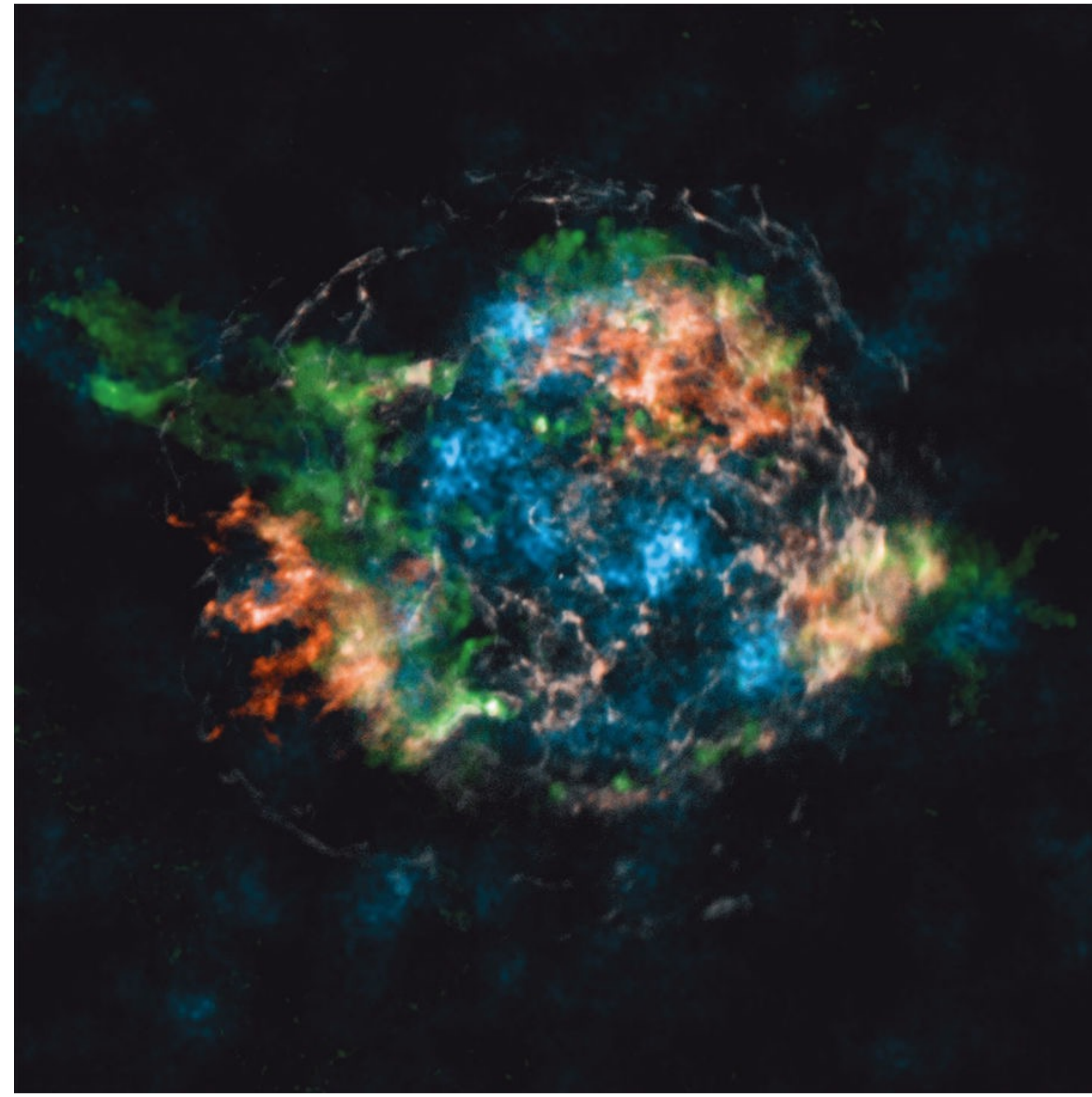
- IR/NIR forbidden lines

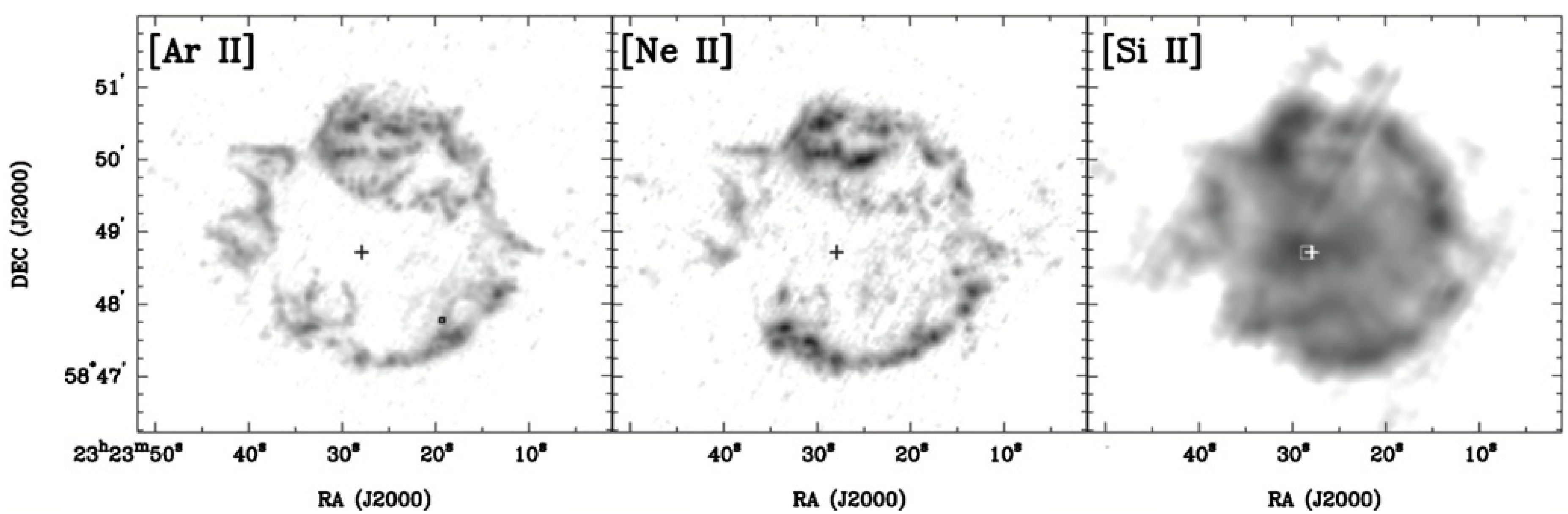




Unshocked ejecta are difficult to probe

- Radioactive decay of elements synthesised in the explosion (^{44}Ti has $t_{1/2}$=60 years) [Grefenstette+ 16]
- IR/NIR forbidden lines
- Cold dust mixed with unshocked ejecta



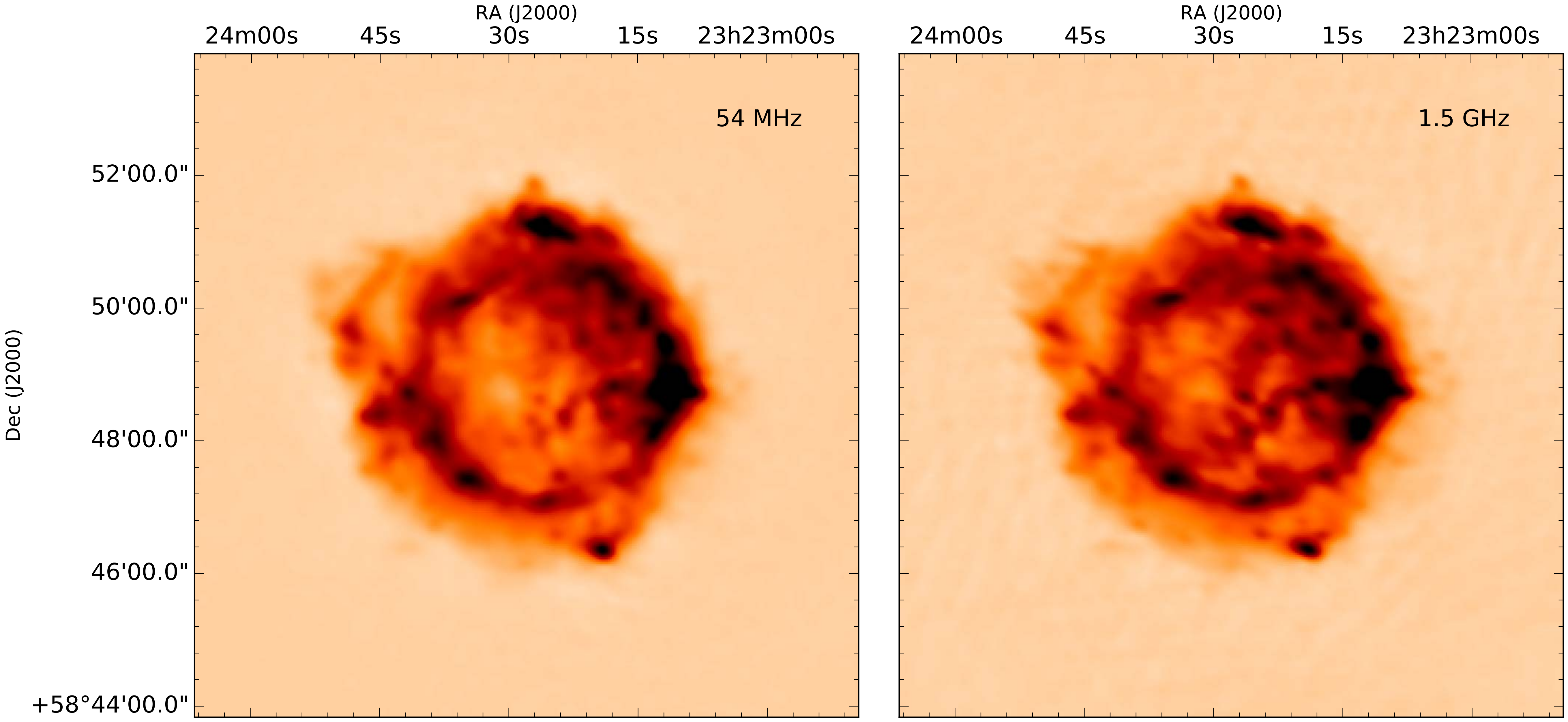


Unshocked ejecta
from low-frequency free-free absorption
[Kassim+ 95]
[Delaney+ 14]

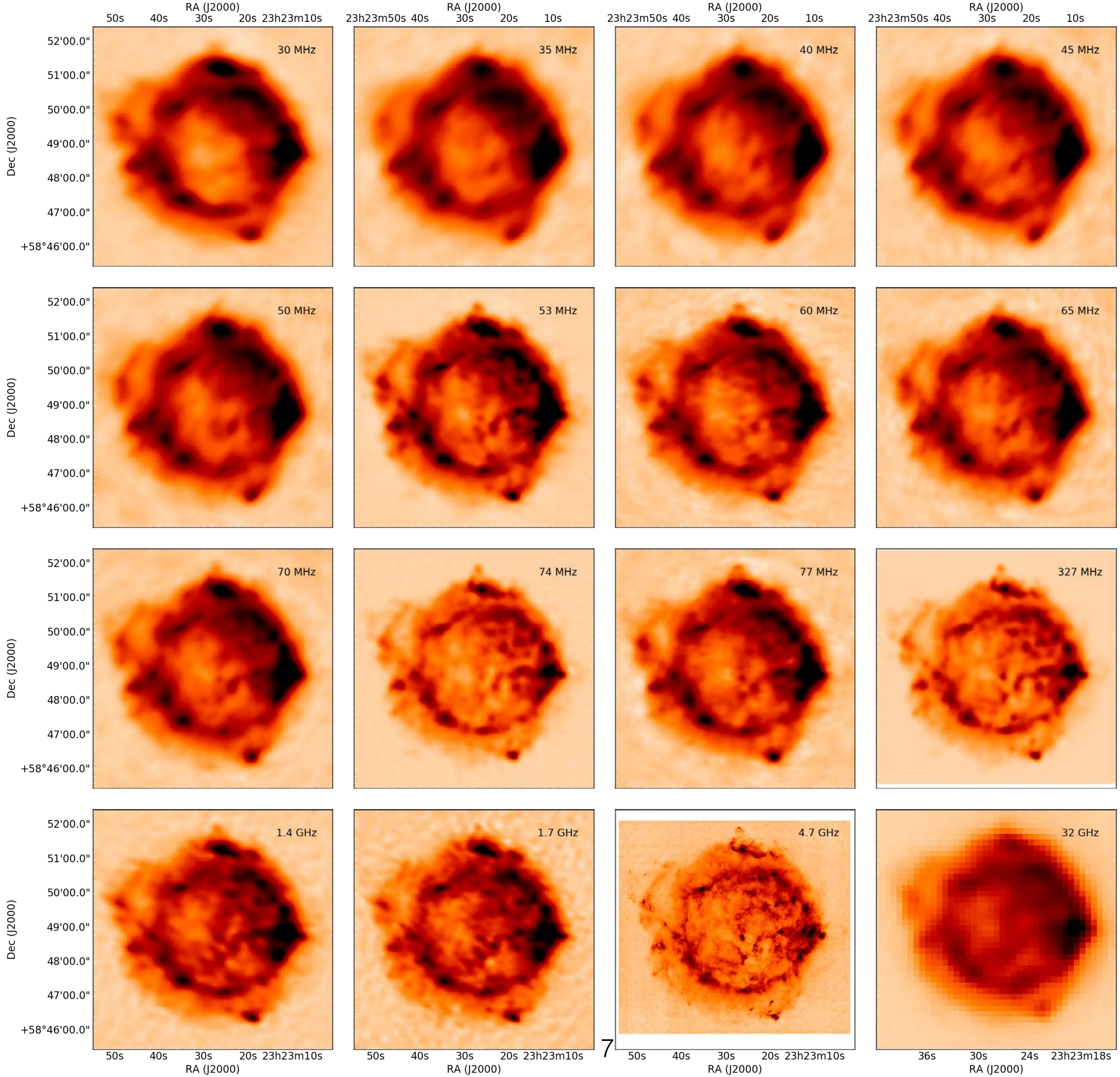
$$S_\nu = (S_{\nu,\mathrm{front}} + S_{\nu,\mathrm{back}} \exp -\tau_{\nu,\mathrm{int}}) \exp -\tau_{\nu,\mathrm{ISM}}$$

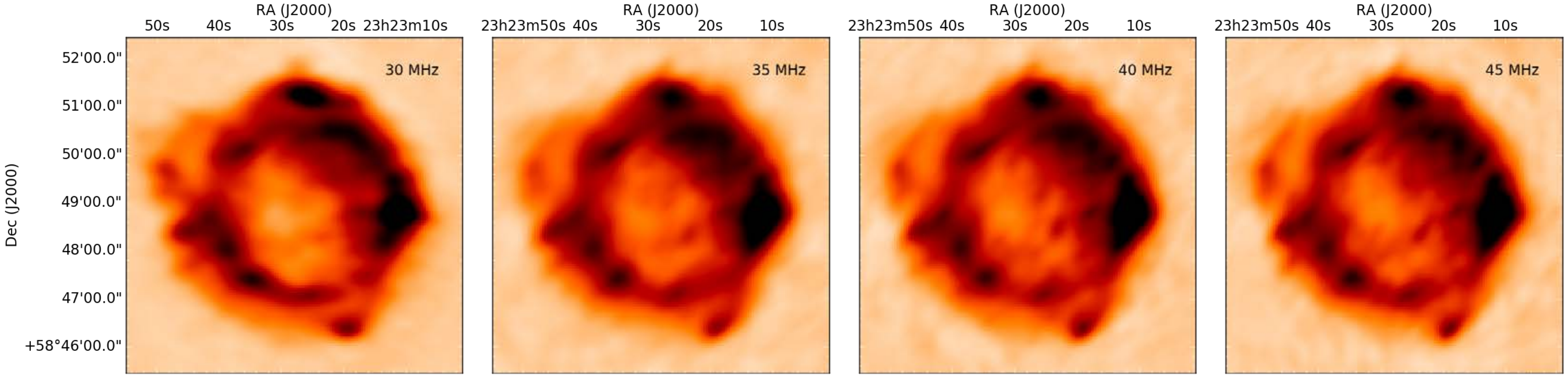
[Arias+ 18]

Cassiopeia A



Cas A as seen with the LOFAR LBA and VLA L-band with10'' resolution. Source size is ~5' [Arias+18]


RA (J2000)
Dec (J2000)
30 MHz
35 MHz
40 MHz
45 MHz
50 MHz
53 MHz
60 MHz
65 MHz
70 MHz
74 MHz
77 MHz
327 MHz
1.4 GHz
1.7 GHz
4.7 GHz
32 GHz



$$S_\nu = S_0 \left(\frac{\nu}{\nu_0}\right)^{-\alpha} \left(f + (1 - f) e^{-\tau_{\nu,\mathrm{int}}}\right),$$

Dec (J2000)

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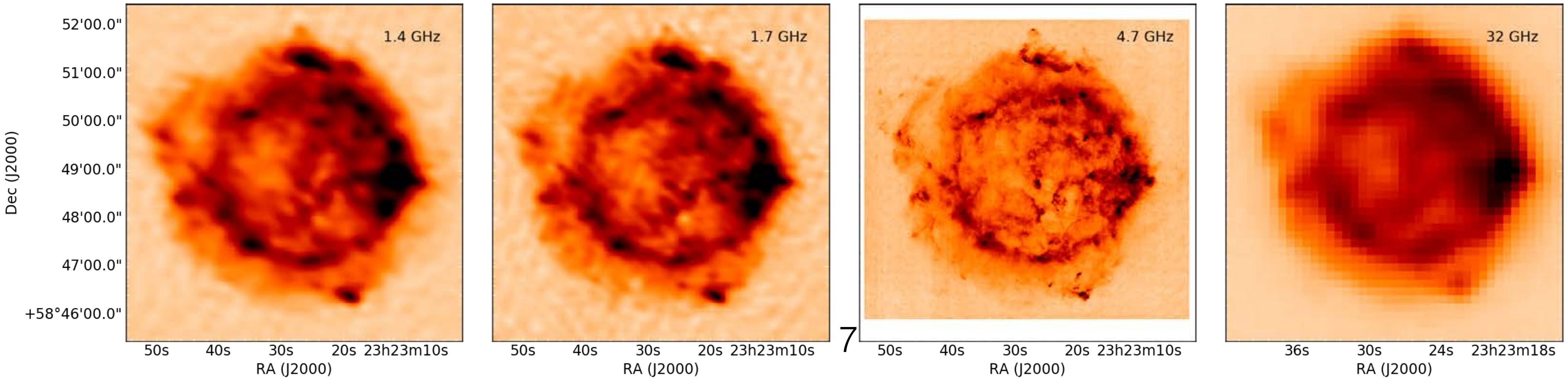
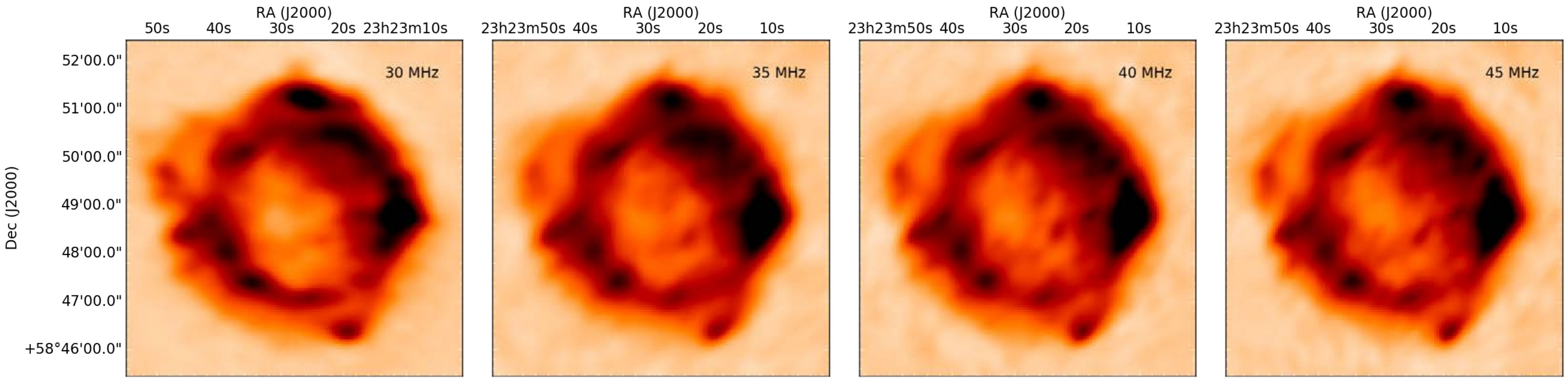
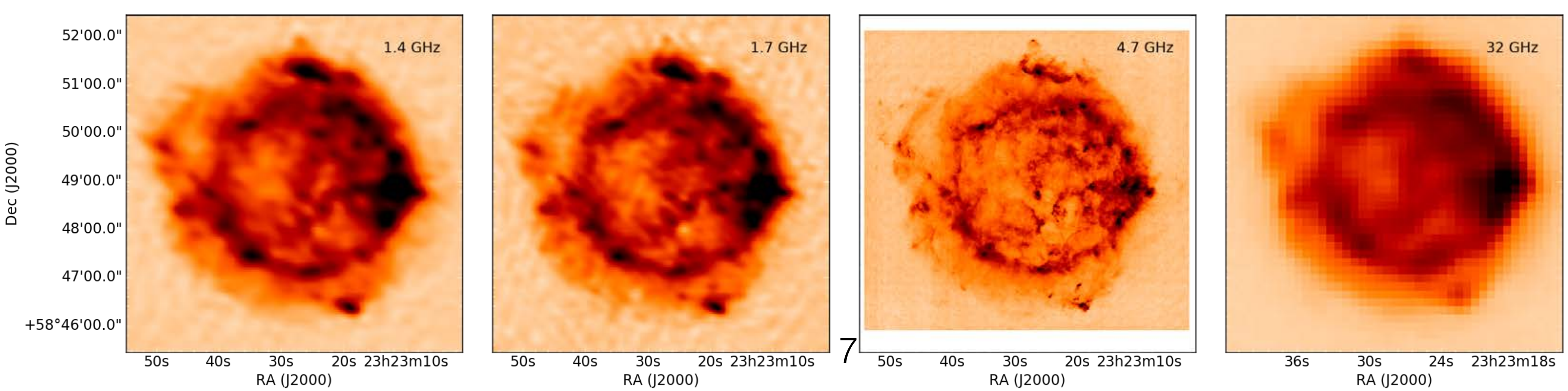


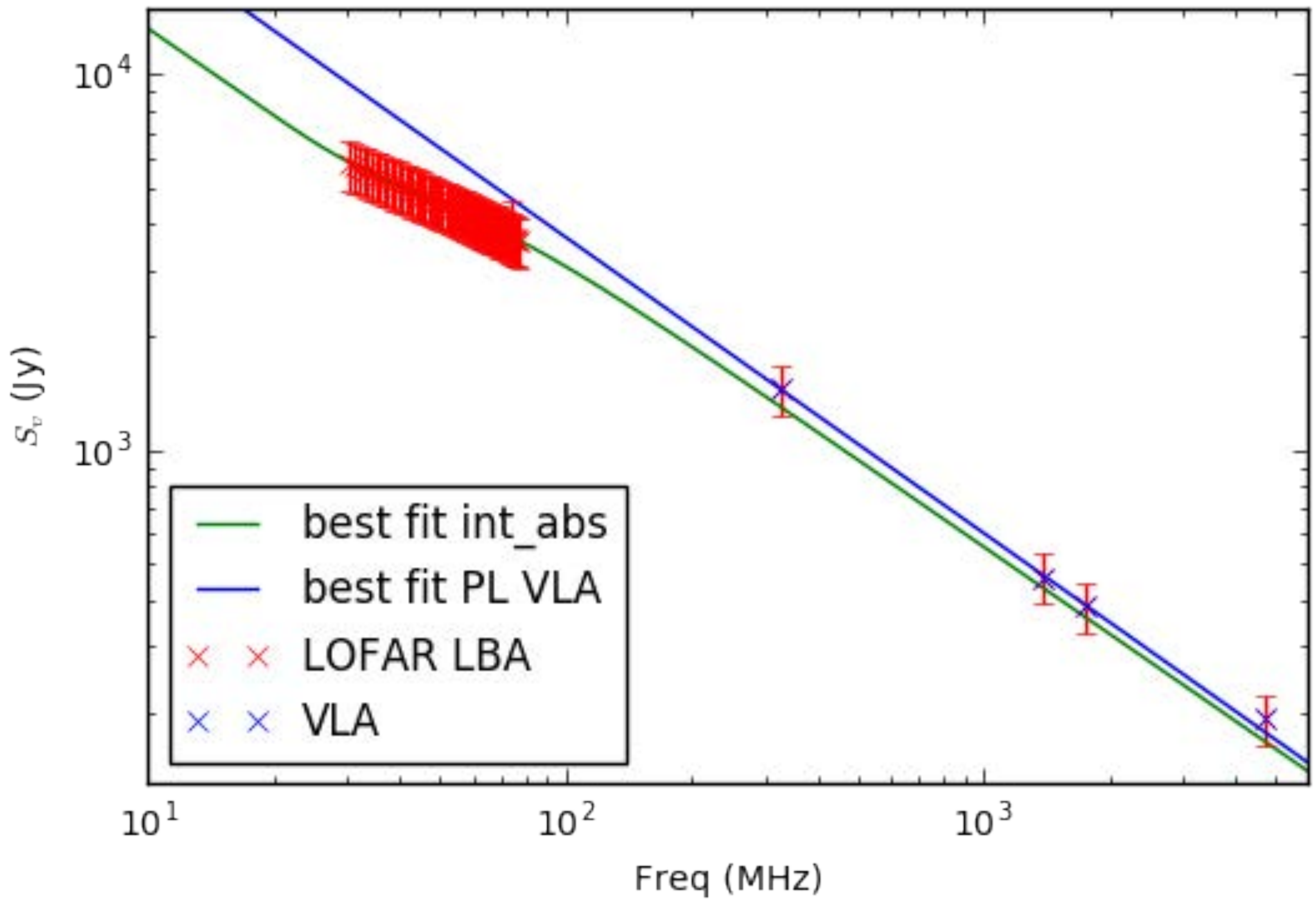


$$S_\nu = S_0 \left(\frac{\nu}{\nu_0}\right)^{-\alpha} \left(f + (1-f)e^{-\tau_{\nu,\mathrm{int}}}\right),$$

where

$$\tau_\nu = 3.014 \times 10^4 \, Z \, \left(\frac{T}{\mathrm{K}}\right)^{-3/2} \left(\frac{\nu}{\mathrm{MHz}}\right)^{-2} \left(\frac{EM}{\mathrm{pc\,cm^{-6}}}\right) g_{\mathrm{ff}}$$





$$S_\nu = S_{1\mathrm{GHz}} \left(\frac{\nu}{1\mathrm{GHz}}\right)^{-0.77} \left(f + (1-f)e^{-\tau_{\nu,\mathrm{int}}}\right)$$

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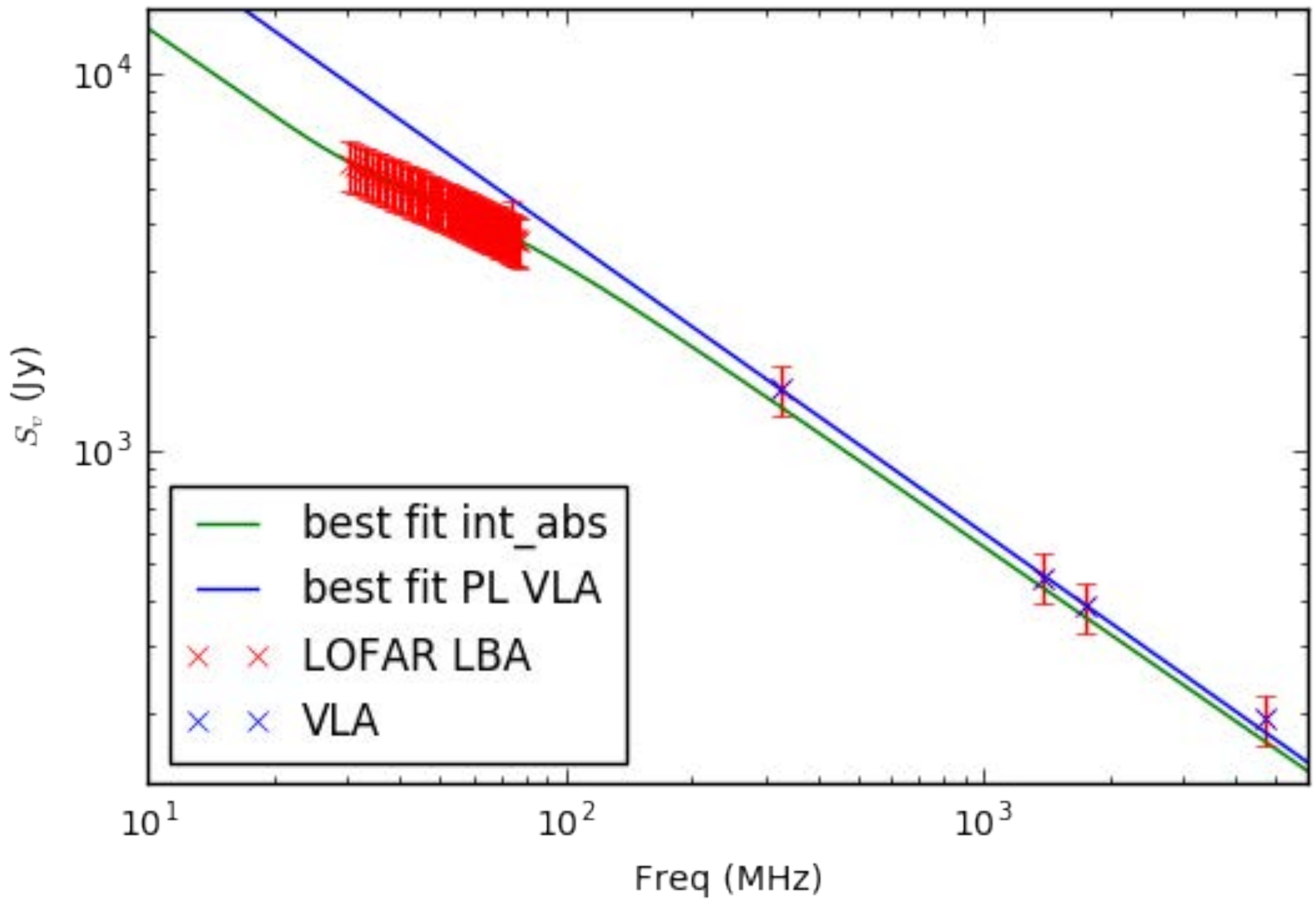

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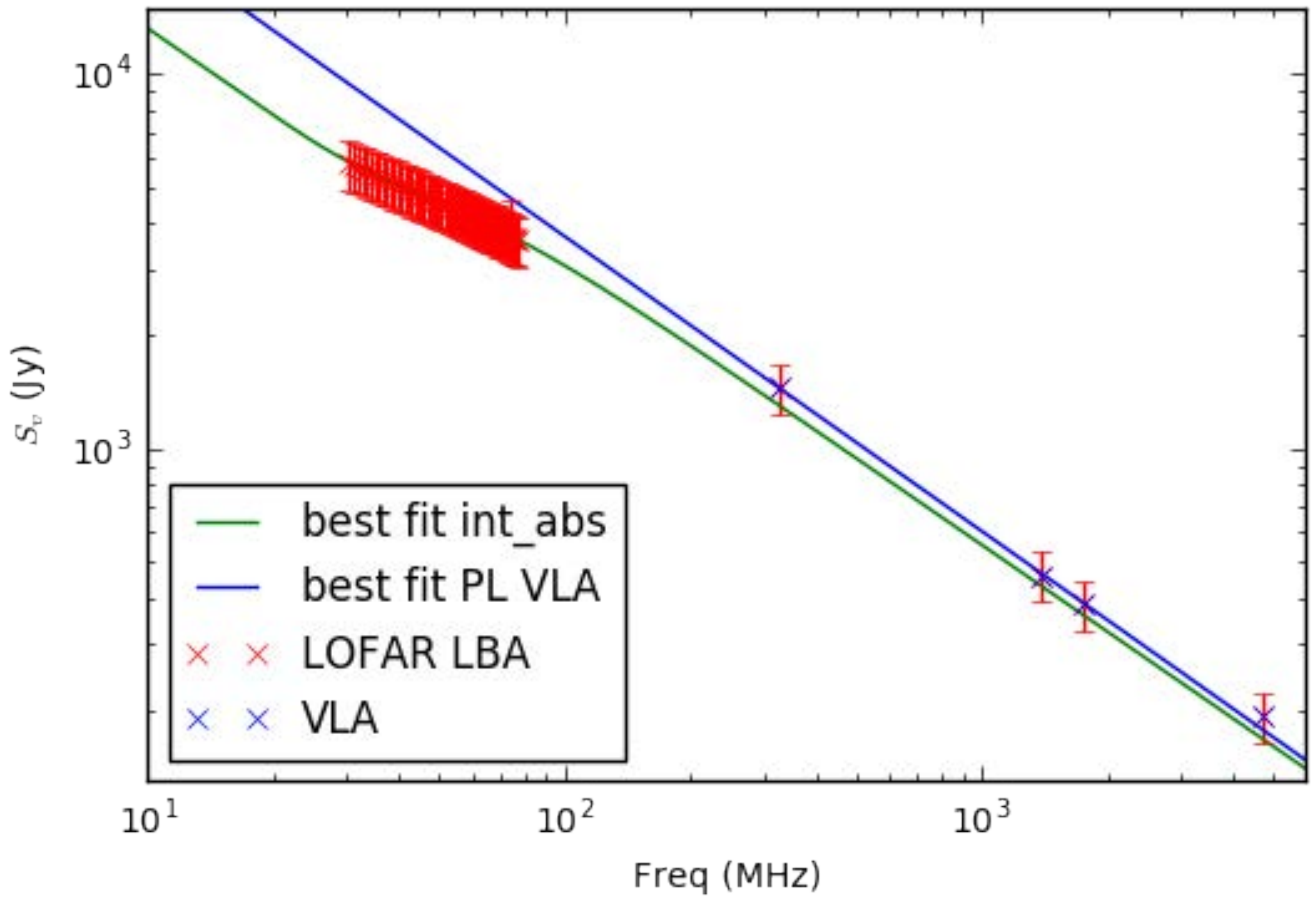

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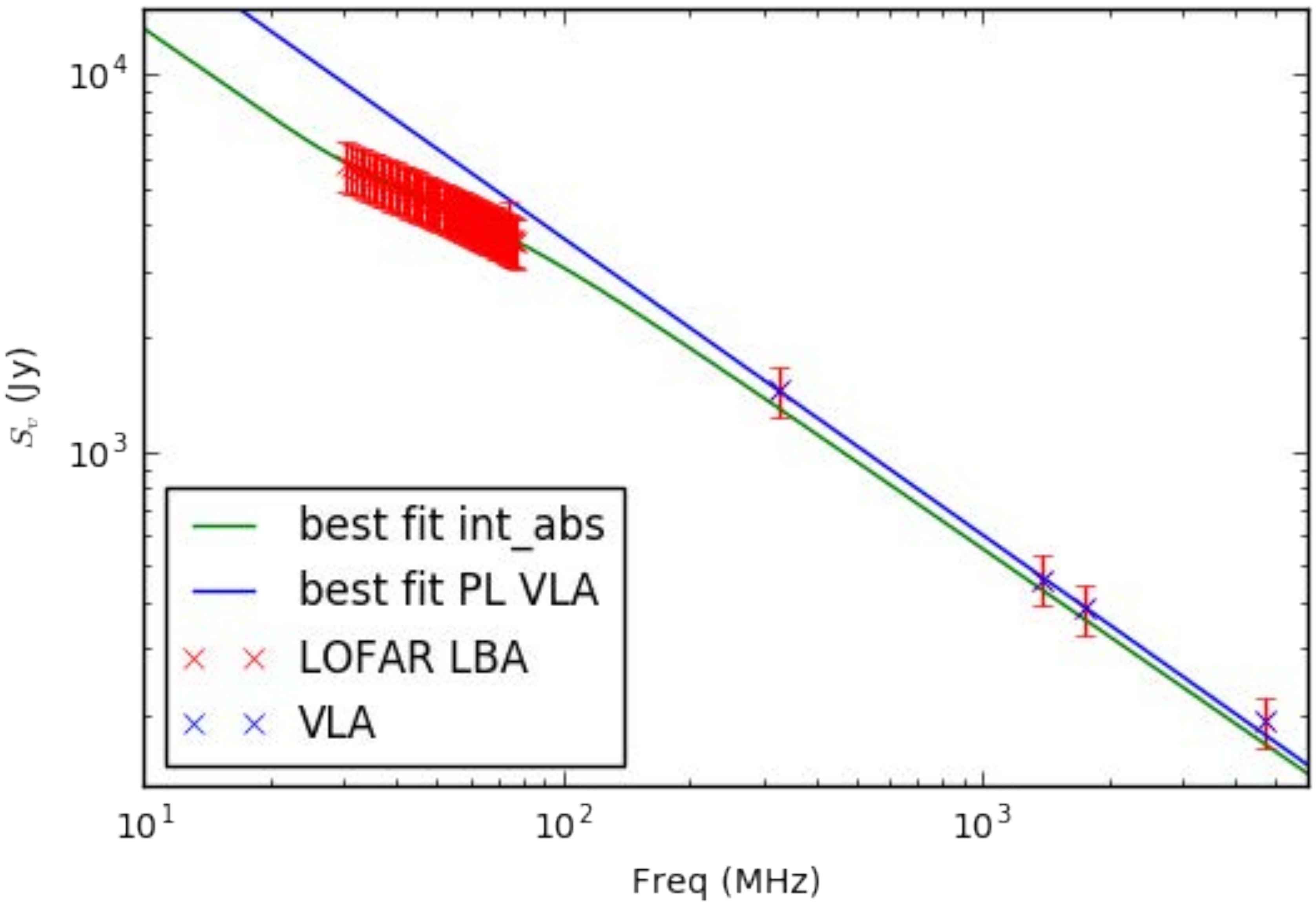

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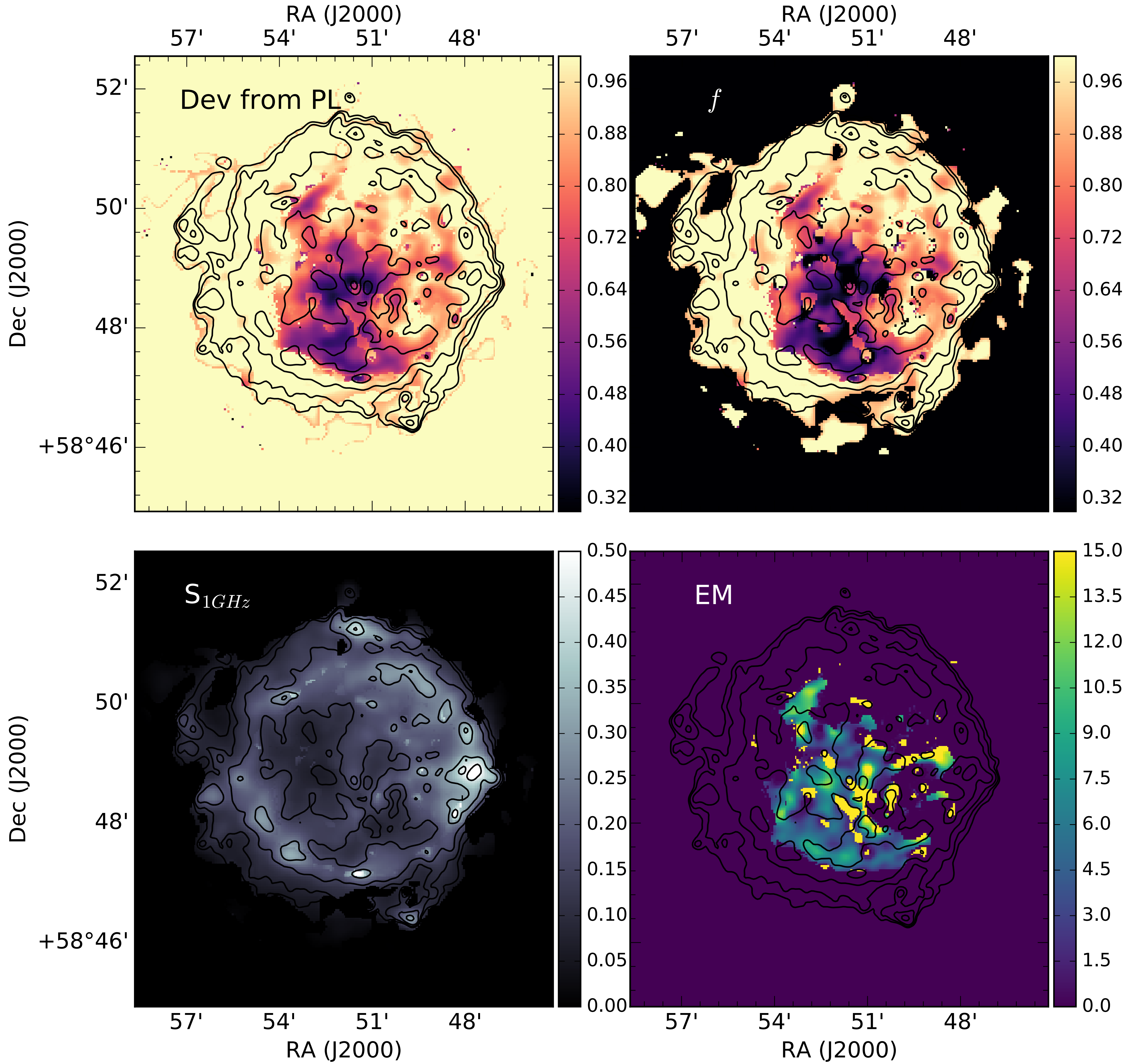

Dev from PL
f
S$_{1GHz}$
EM
RA (J2000)
Dec (J2000)
57'
54'
51'
48'
52'
50'
48'
+58°46'
0.96
0.88
0.80
0.72
0.64
0.56
0.48
0.40
0.32
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
15.0
13.5
12.0
10.5
9.0
7.5
6.0
4.5
3.0
1.5
0.0

Mass is a function of a lot of things we don't know

$$M = ASl^{1/2} m_{\rm p} \frac{1}{Z} \sqrt{EM}$$

- Temperature
- Charges (ionisation)
- Composition
- Geometry

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- Temperature 100 K (Raymond+18)
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- Temperature 100 K (Raymond+18)
- Charges (ionisation) low ionisation species [Si II] (Smith+ 09), [O IV] (Isensee+ 10), [S III] (Milisavljevic & Fesen 15)
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- Composition Si
  O (Delaney+ 14)
  S, Fe? (Milisavljevic & Fesen 15)
- Geometry

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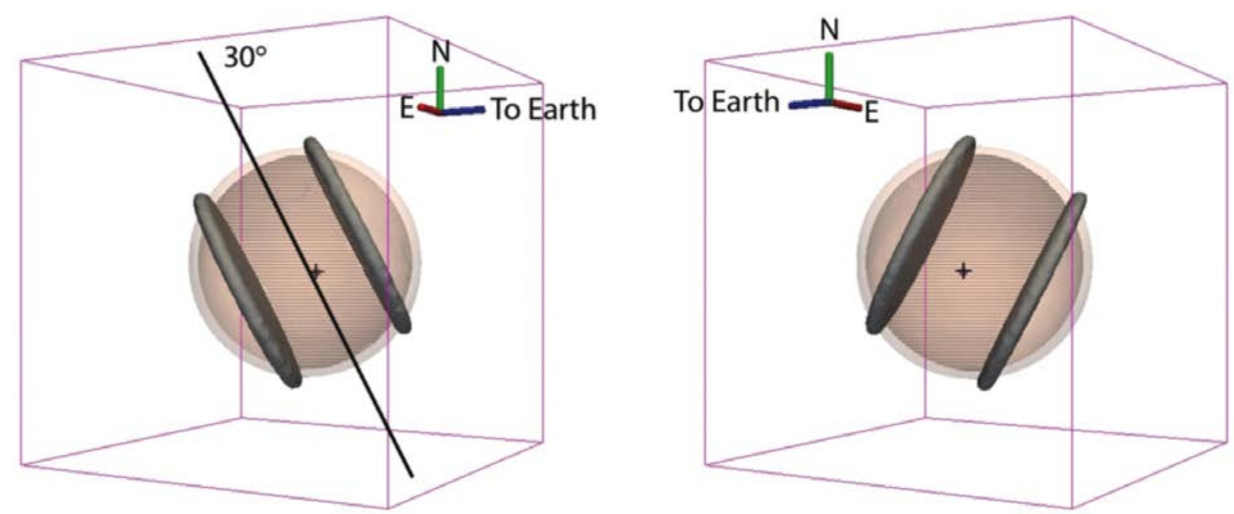
- Geometry

Geometry

- Thick disk [Delaney+ 10, Isensee+ 10]
- Cavities or bubbles [Milisavljevic & Fesen 15]
- Diffuse emission, with clumps, filaments, arcs [Koo+ 18]



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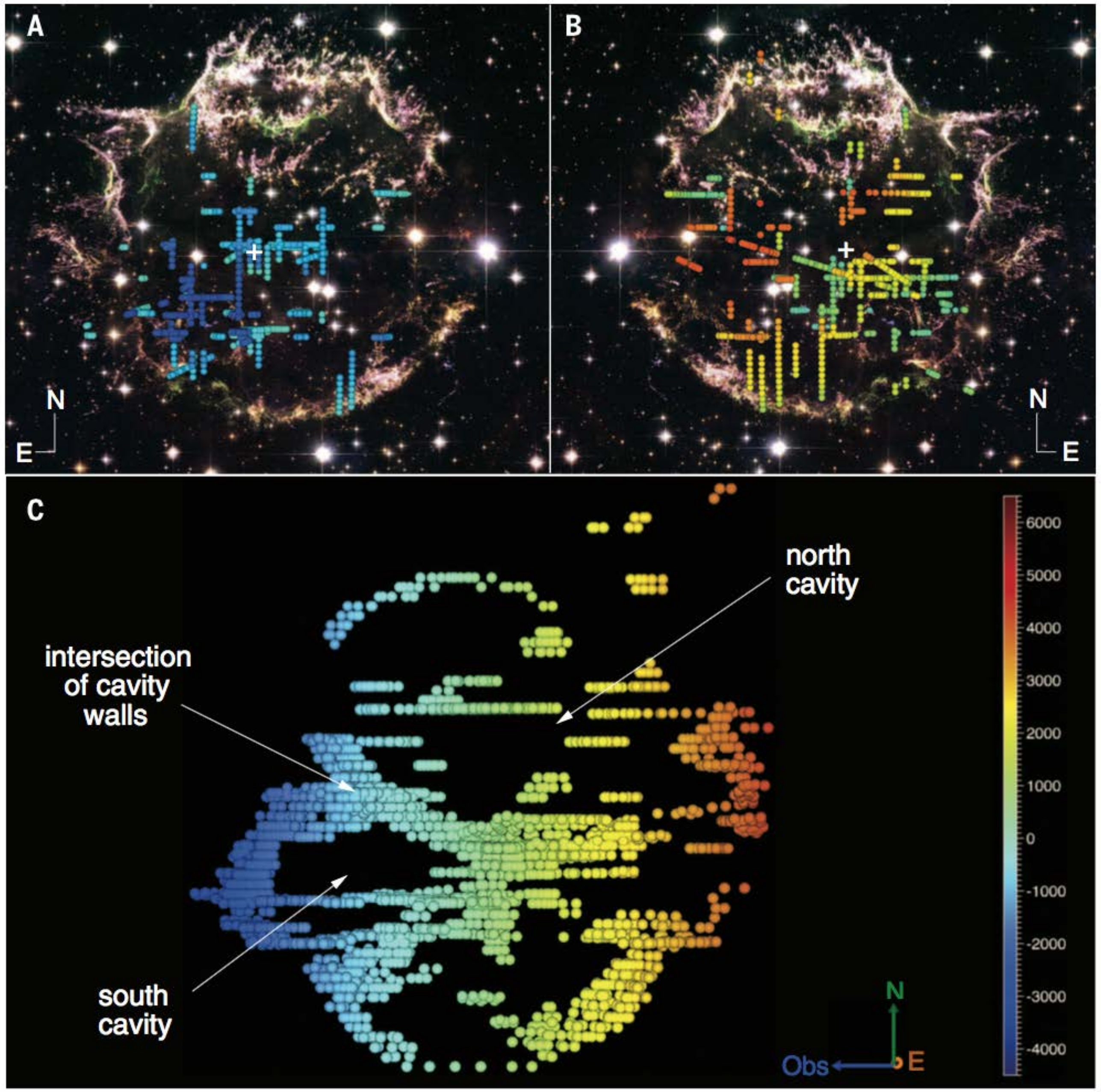
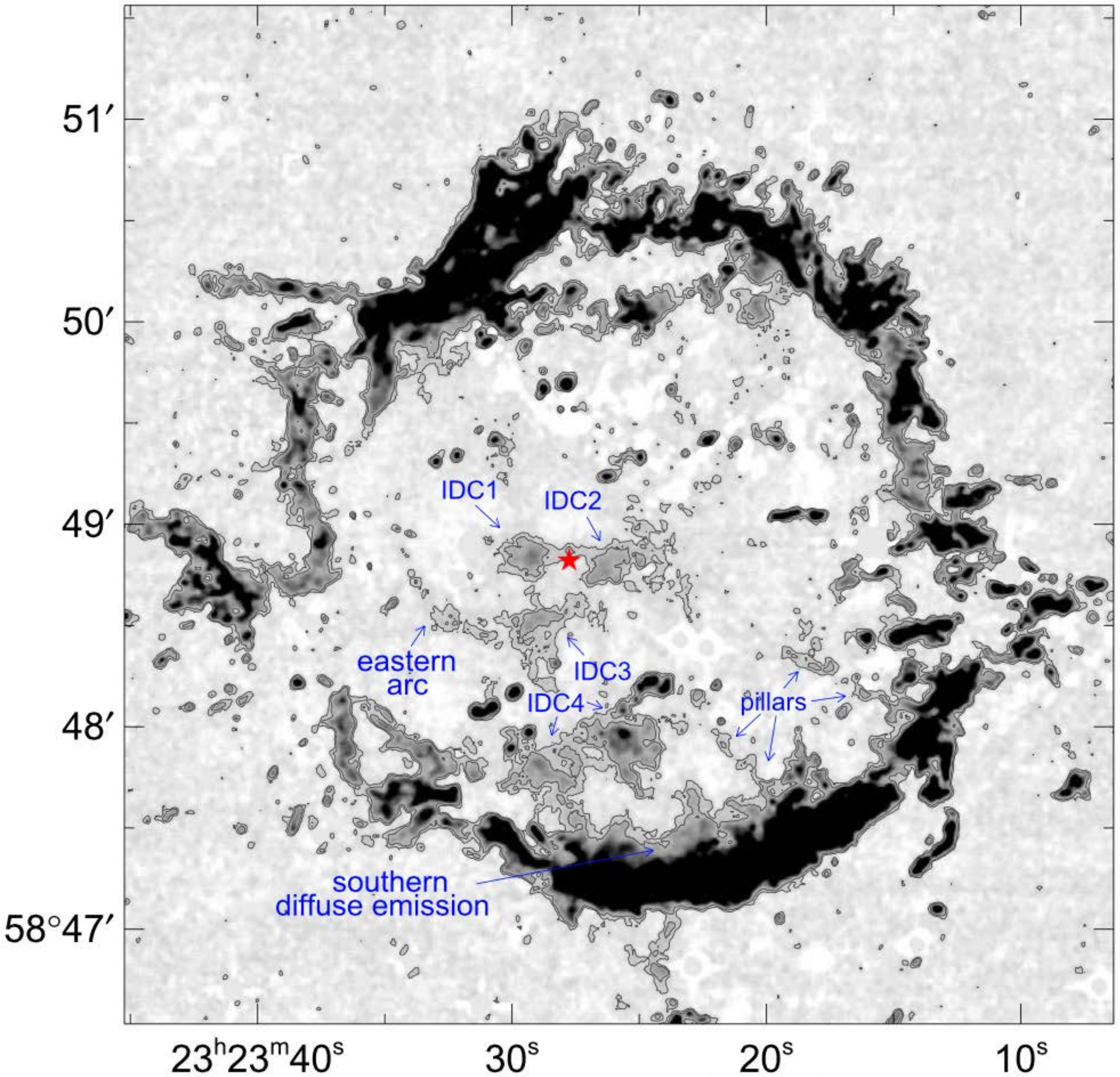
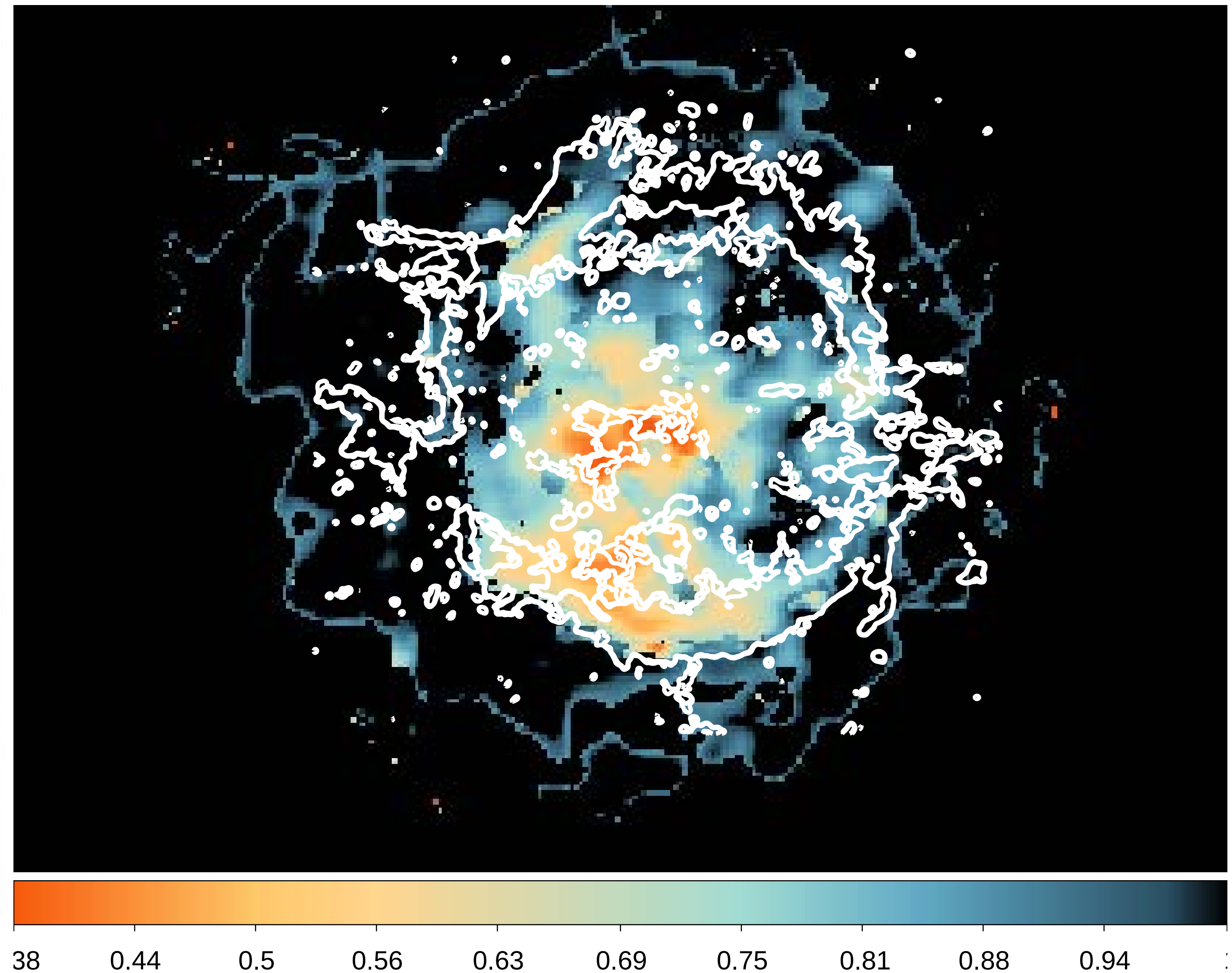




deep [Fe II]+[Si I] image
Koo+ 18



contours overlaid on our PL
deviation map

$$M = 0.53 \pm 0.10 M_{\odot} \left(\frac{A}{16}\right)\left(\frac{l}{0.16\mathrm{pc}}\right)^{1/2}\left(\frac{Z}{3}\right)^{-3/2}\left(\frac{T}{100\ \mathrm{K}}\right)^{3/4} \times \sqrt{\frac{g_{\mathrm{ff}}(T=100\,\mathrm{K}, Z=3)}{g_{\mathrm{ff}}(T,Z)}}$$

(previous 3 M⊙ in Arias+ 18)

BUT:

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(previous 3 M⊙ in Arias+ 18)

BUT:

Oxygen: 16
Silicon: 28
Sulfur: 32
Iron: 55

estimate can increase by factor of 1.5 or 2

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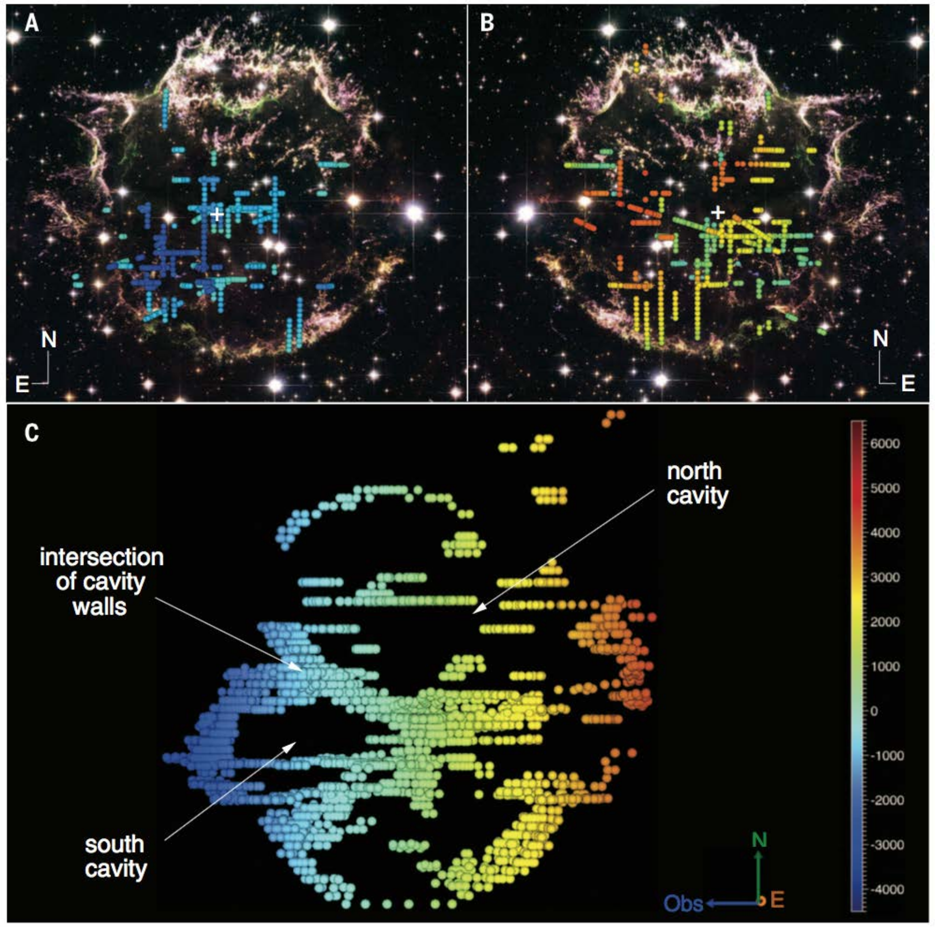
(previous 3 M⊙ in Arias+ 18)

BUT:

Milisavljevic & Fesen 15 has lines-of-sight that are thinner and thicker



can very much alter the estimate

$$M = 0.53 \pm 0.10 M_\odot \left(\frac{A}{16}\right)\left(\frac{l}{0.16\mathrm{pc}}\right)^{1/2}\left(\frac{Z}{3}\right)^{-3/2}\left(\frac{T}{100\ \mathrm{K}}\right)^{3/4} \times \sqrt{\frac{g_{\mathrm{ff}}(T=100\,\mathrm{K}, Z=3)}{g_{\mathrm{ff}}(T,Z)}}$$

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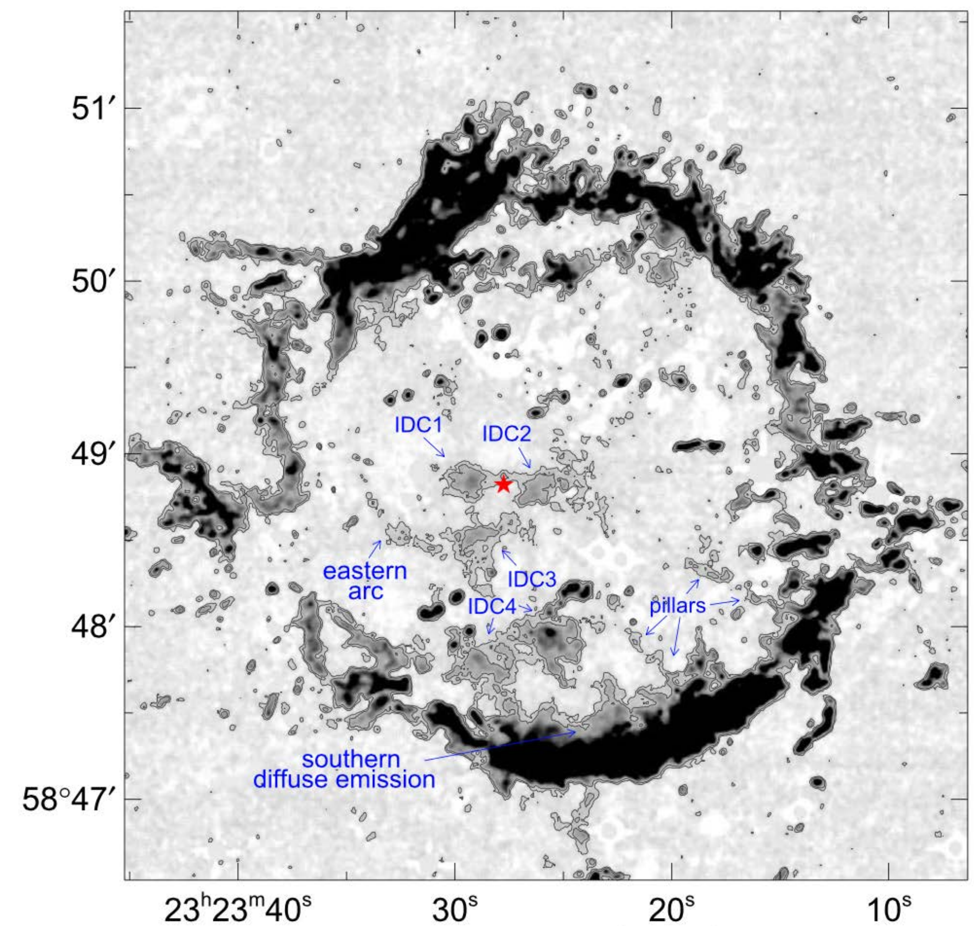
(previous 3 M⊙ in Arias+ 18)

BUT:

Most detected lines of lower ionisations [Si II] (Smith+ 09),[Si I] (Koo+ 18), [O IV] (Isensee+ 10), [S III] (Milisavljevic & Fesen 15)



neutrals not factored

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(previous 3 M⊙ in Arias+ 18)

BUT:

plus, effect of clumping

(Arias+ 18)

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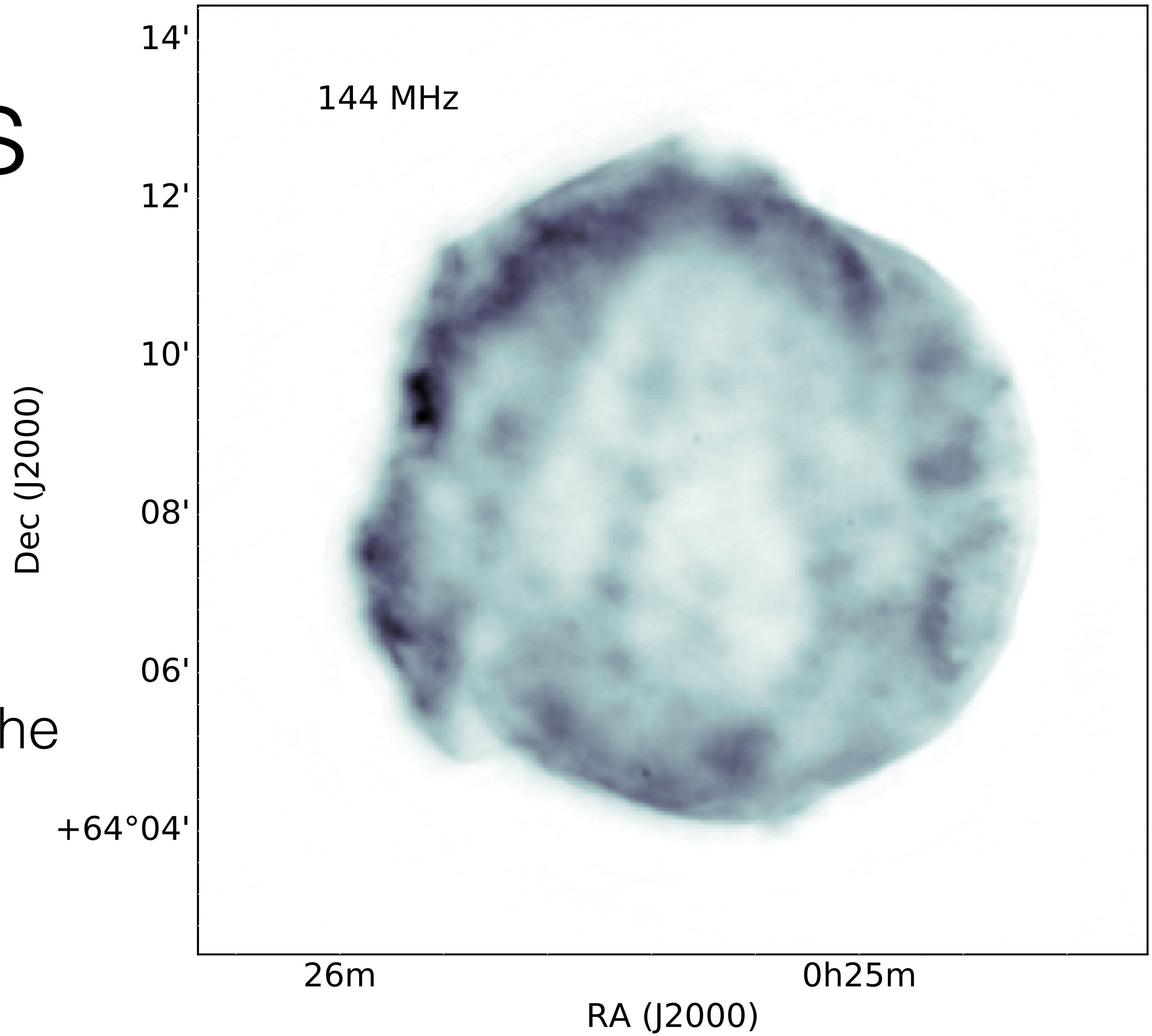
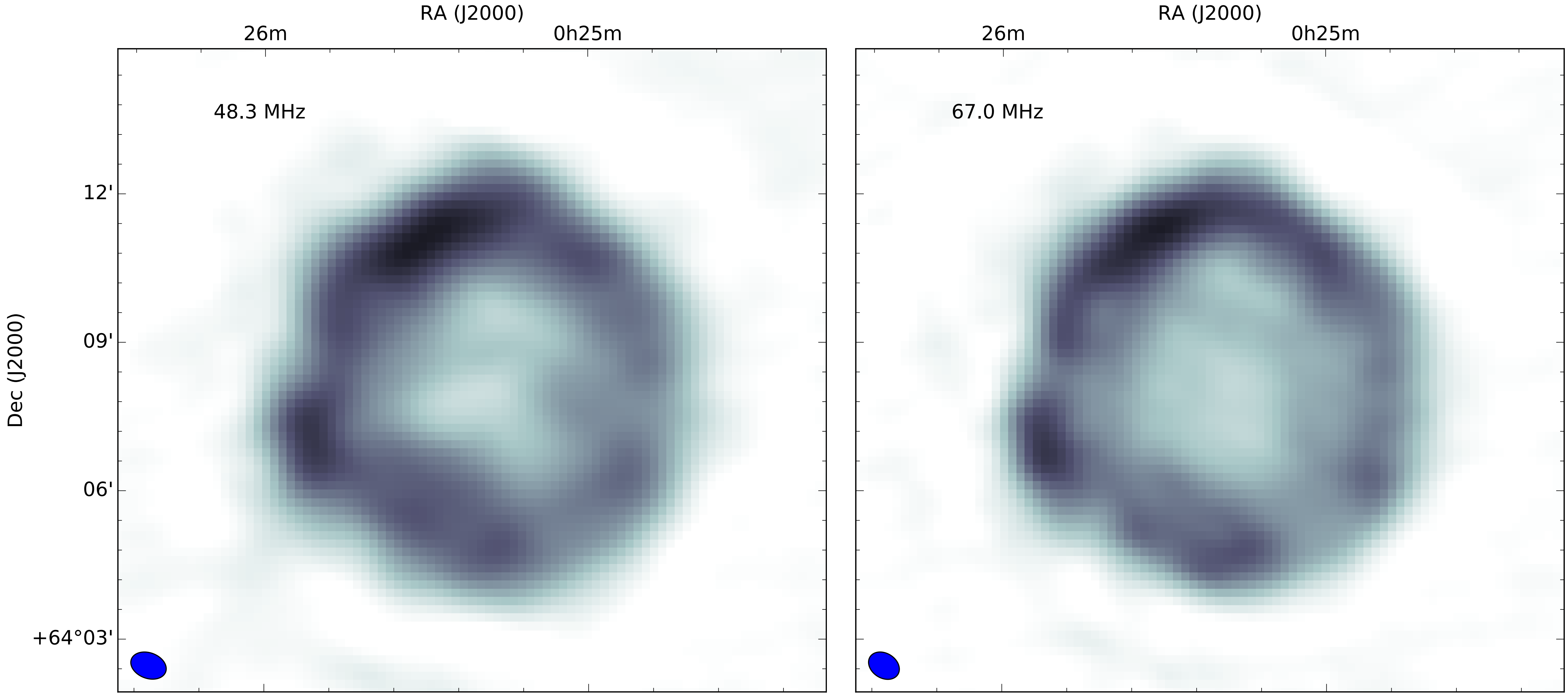
This method allows us to locate the rim of absorbing material (i.e,. the reverse shock) More than providing a firm estimate of the mass, it is a probe into the conditions in the unshocked ejecta

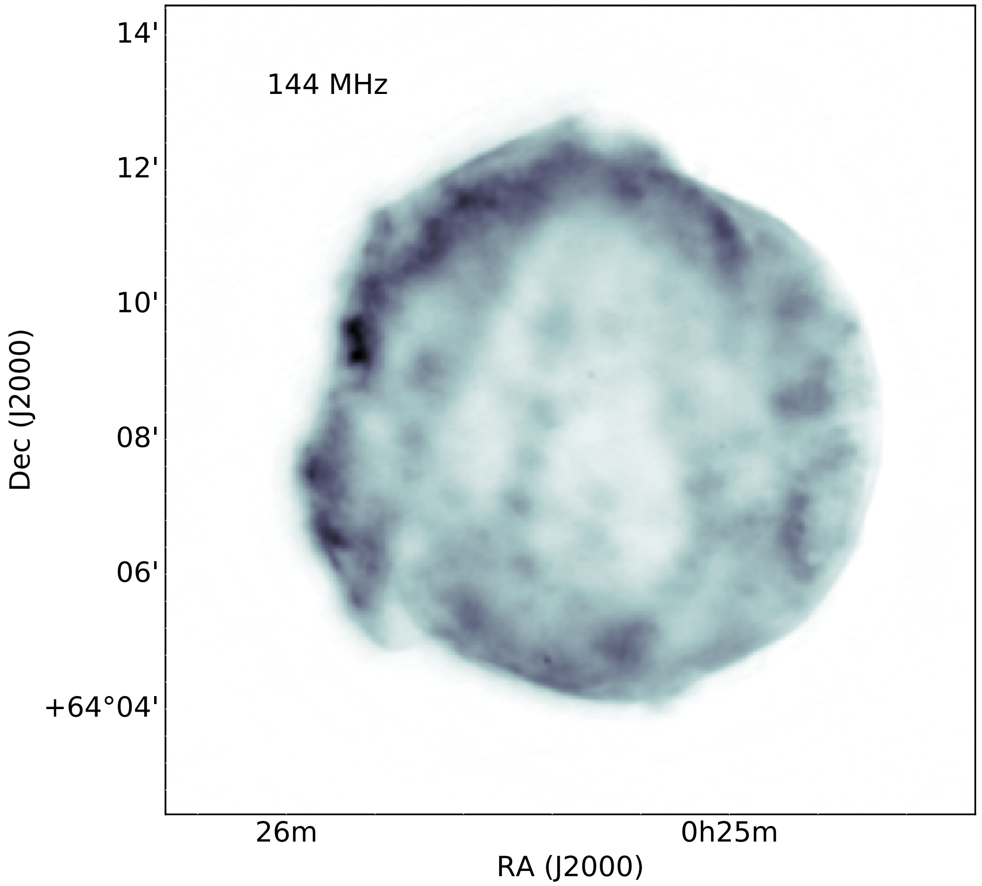
Tycho's SNR



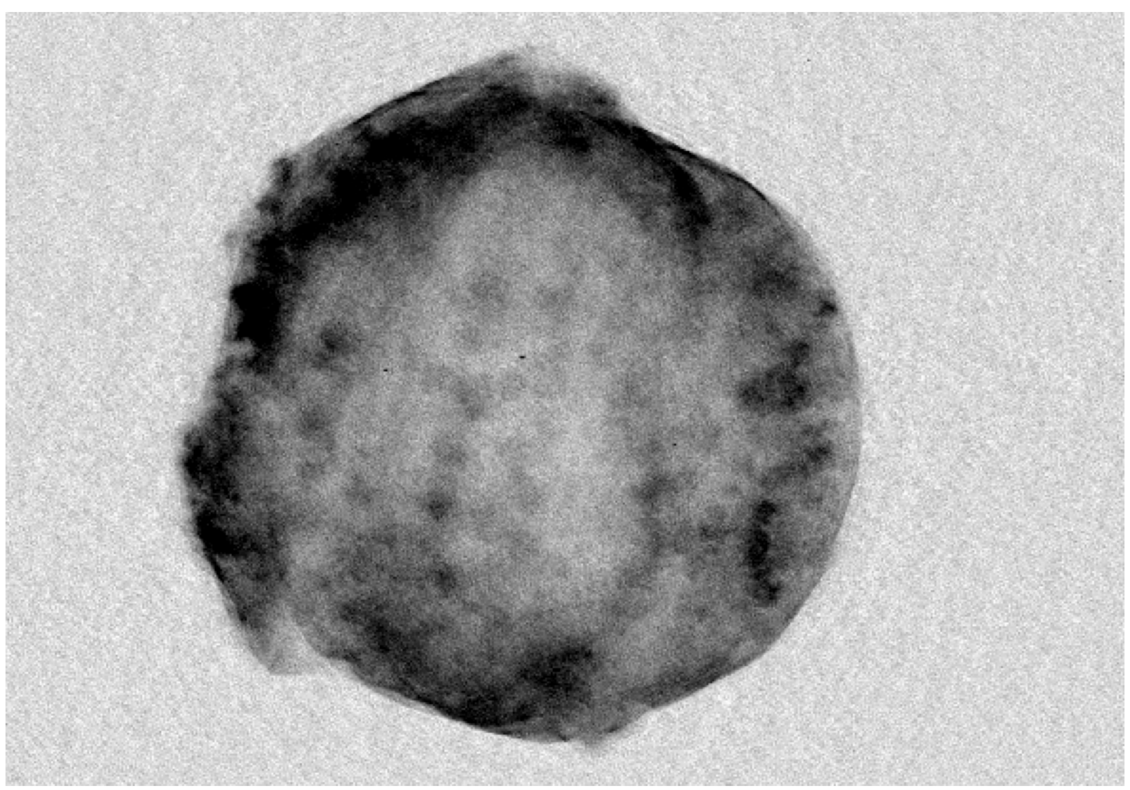
Tycho's SNR in the LOFAR HBA at 143 MHz, 9.5'' resolution



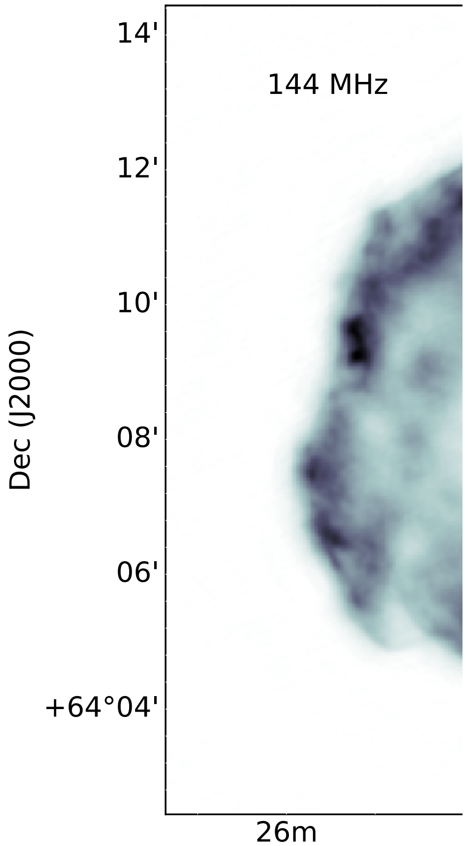
Tycho as seen with the LOFAR LBA (48.3 MHz and 67.0 MHz) with 30'' resolution. Source size is ~8'   Arias+18

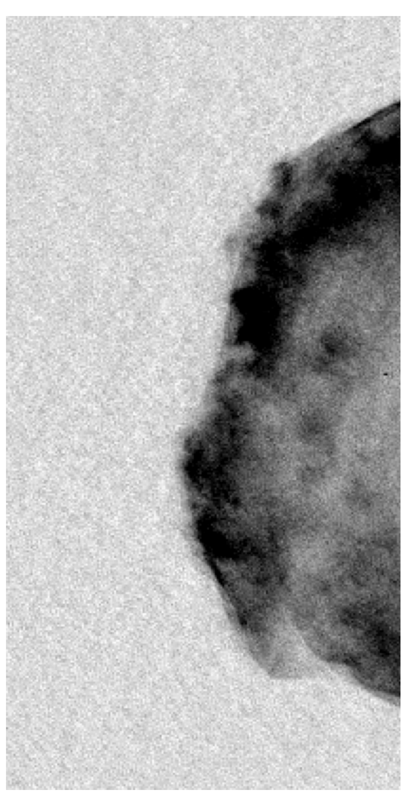


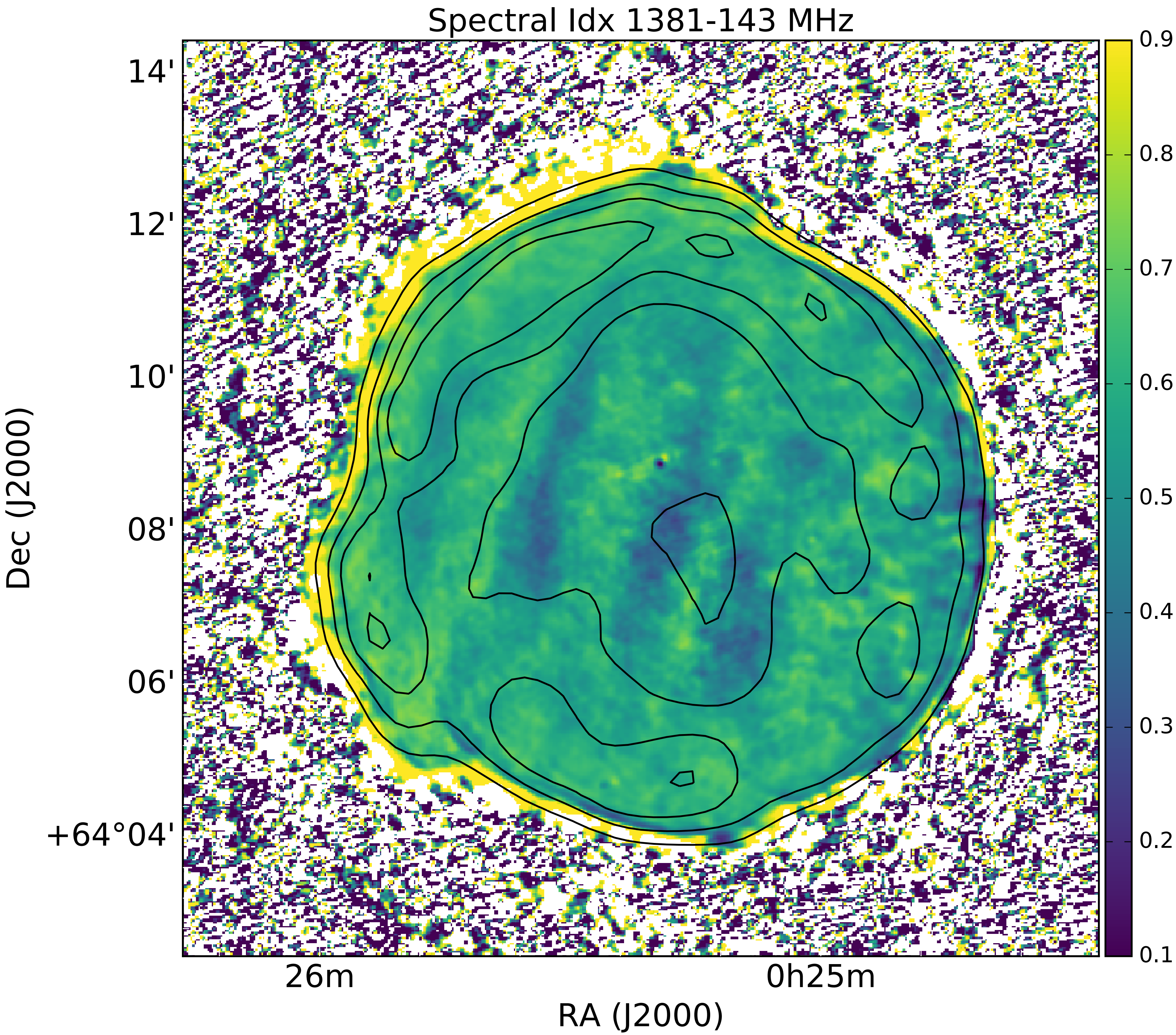
LOFAR HBA
143 MHz



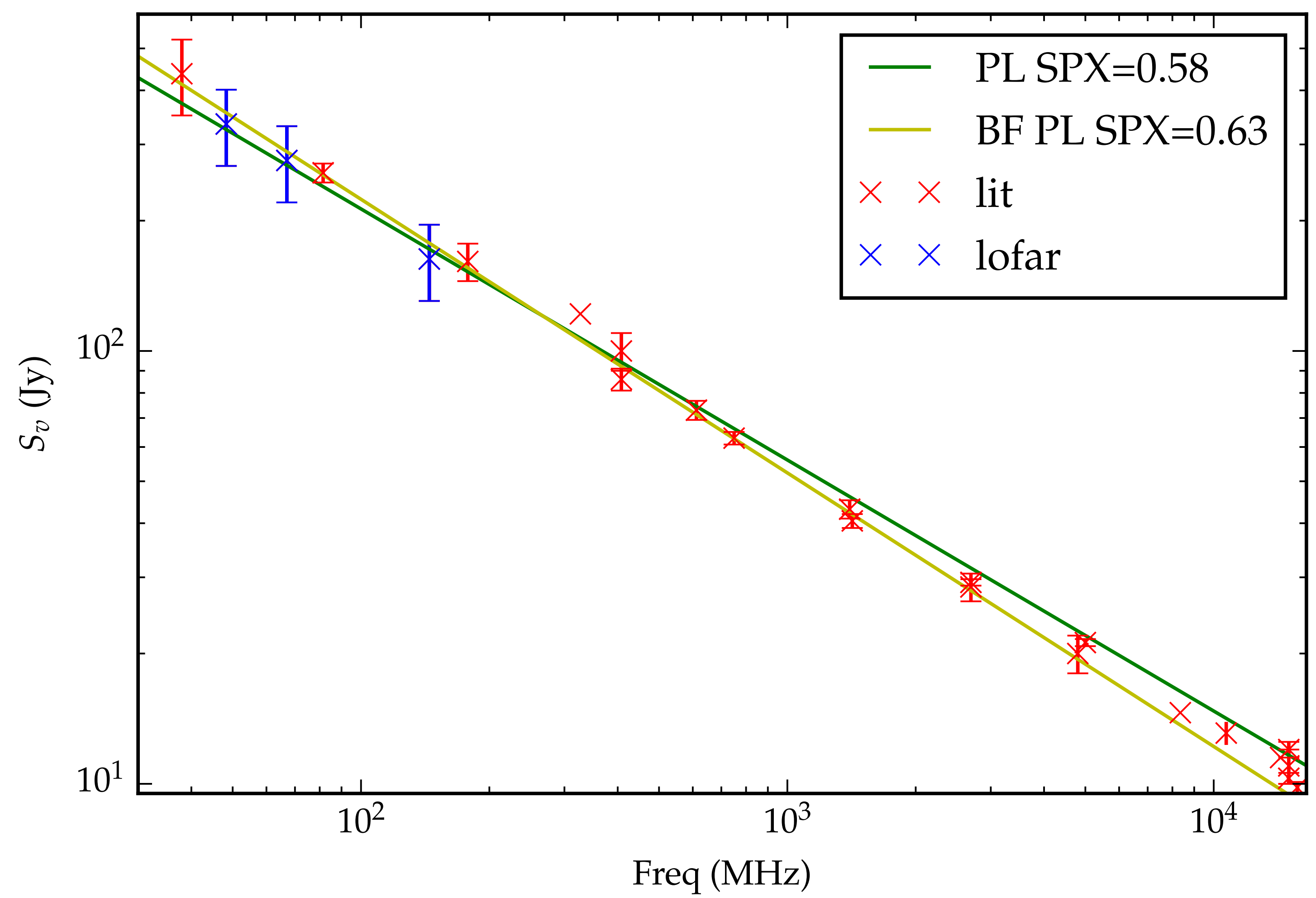
VLA 1.4 GHz
Williams+ 16

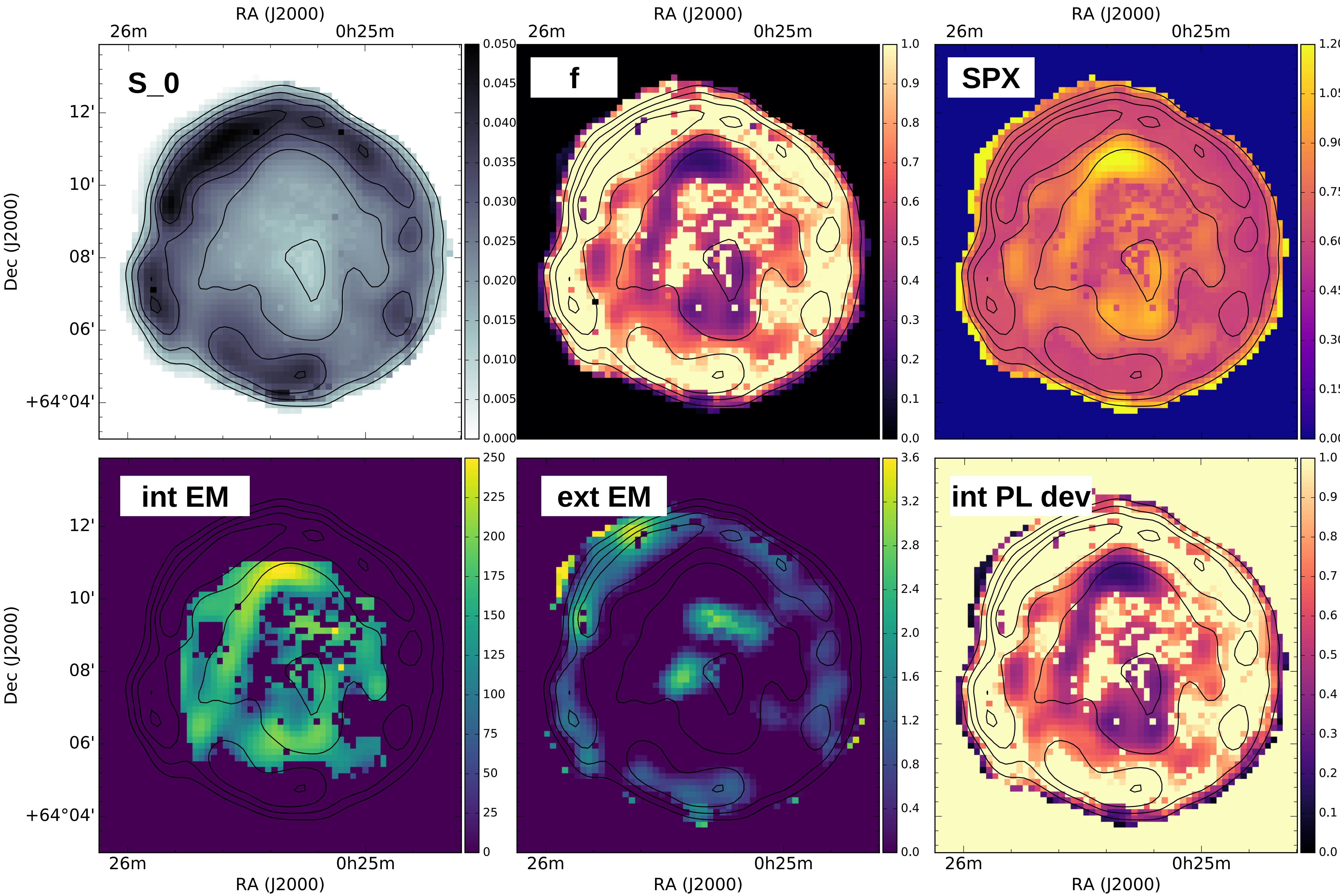

144 MHz
14'
12'
10'
08'
06'
+64°04'
Dec (J2000)
26m




Spectral Idx 1381-143 MHz
14'
12'
10'
08'
06'
+64°04'
Dec (J2000)
26m
0h25m
RA (J2000)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1

Flux densities measured with the LOFAR HBA and LBA match literature values of this well-known source (3C10)





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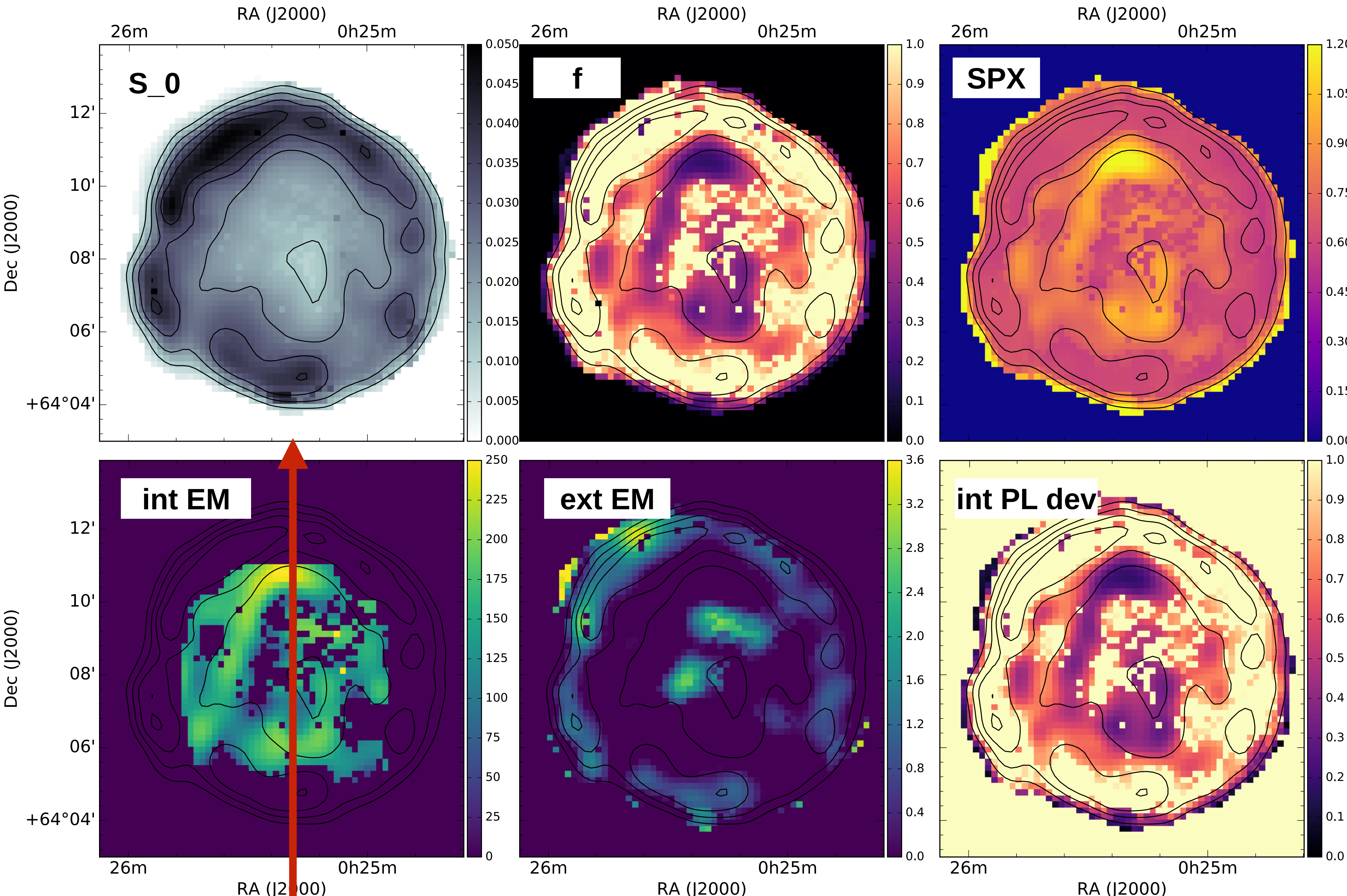

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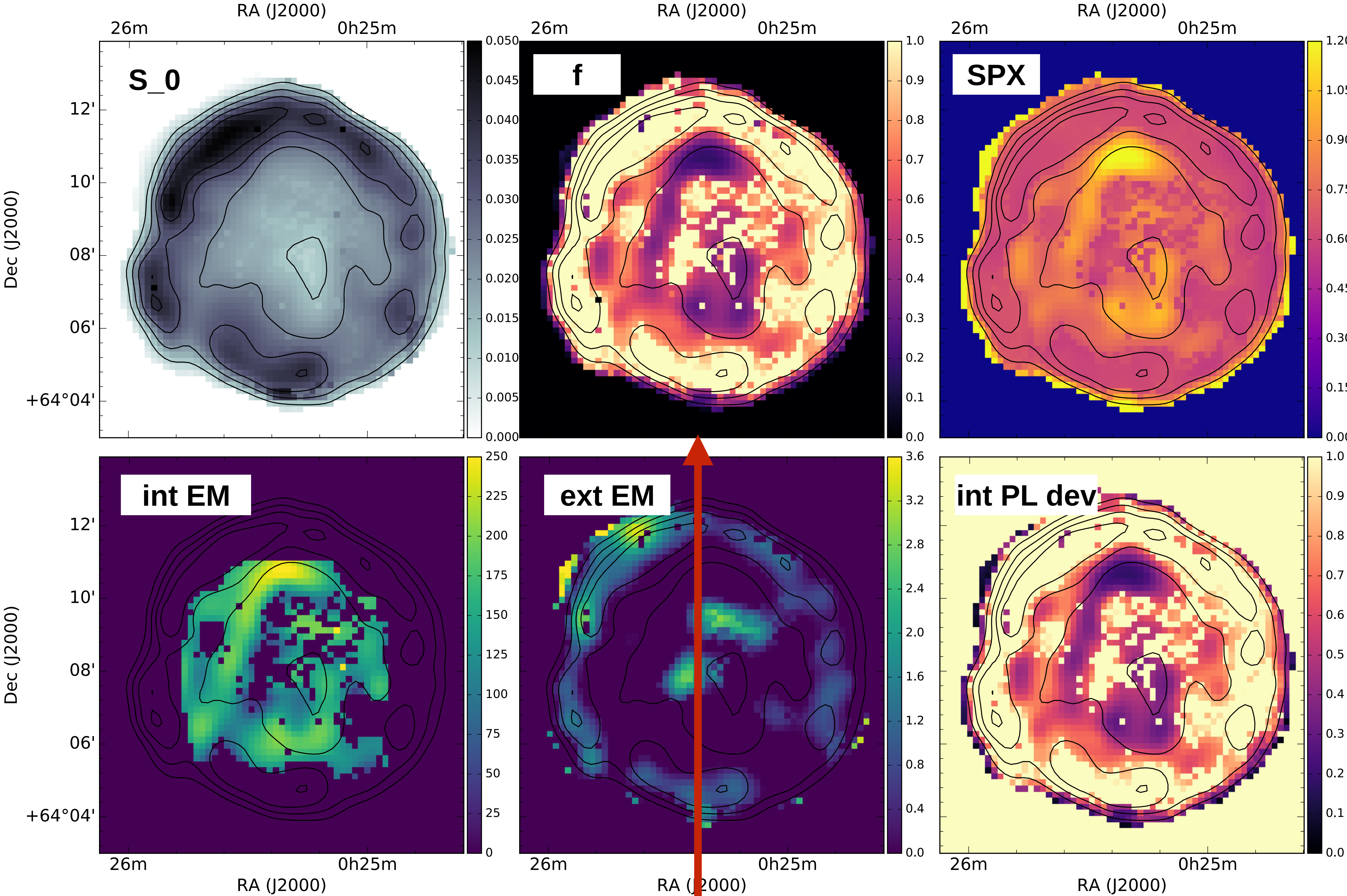

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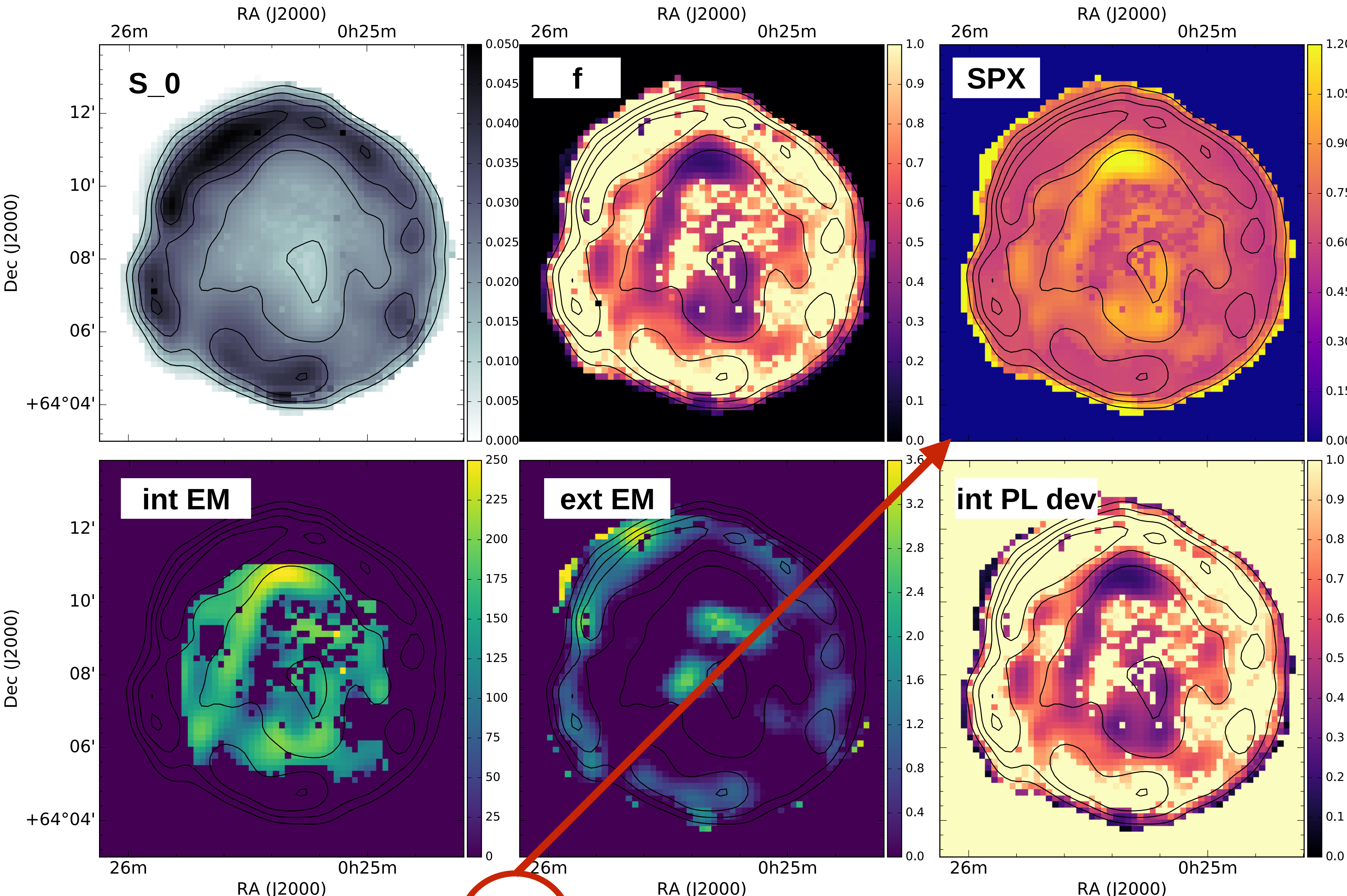

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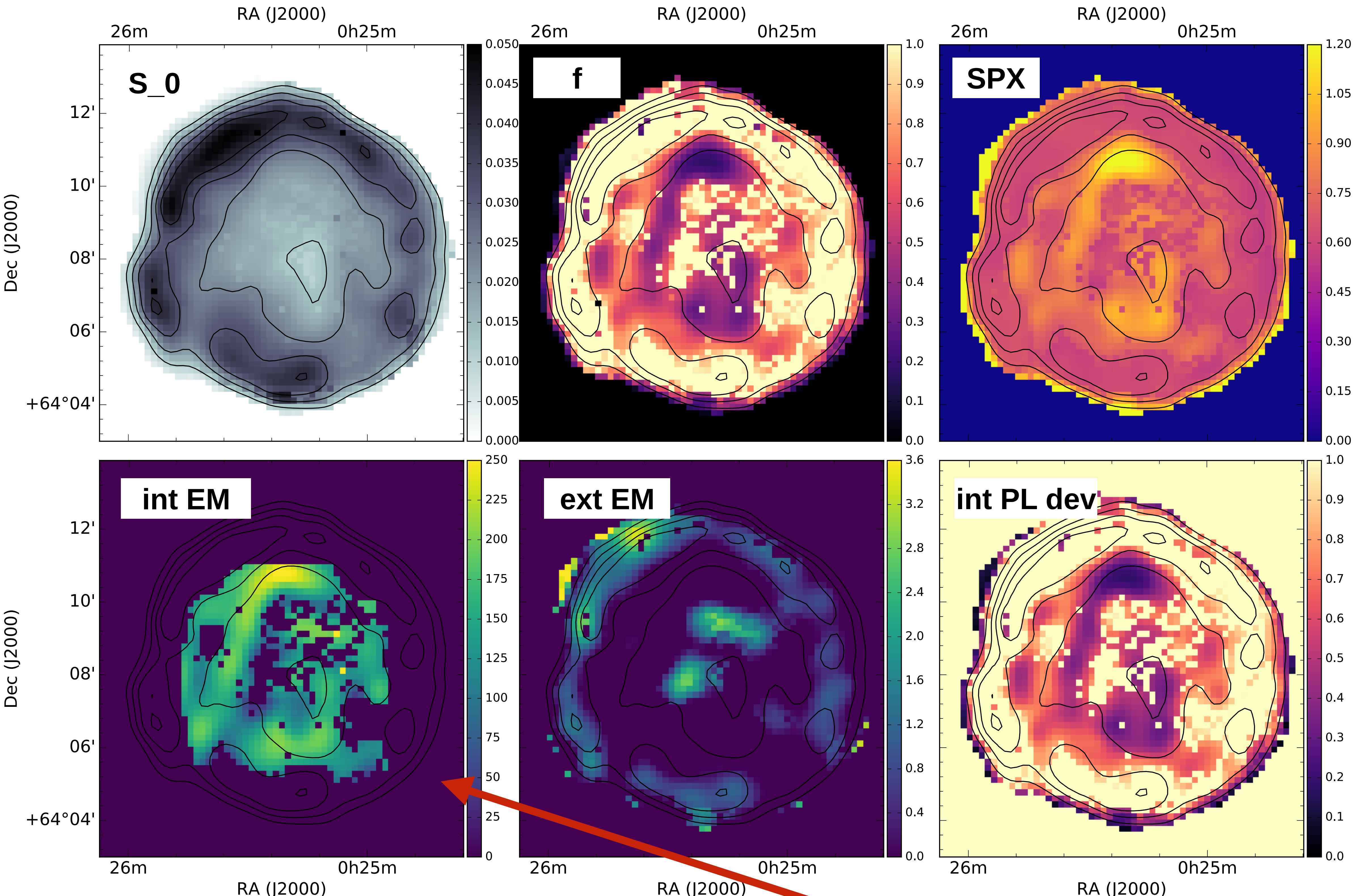

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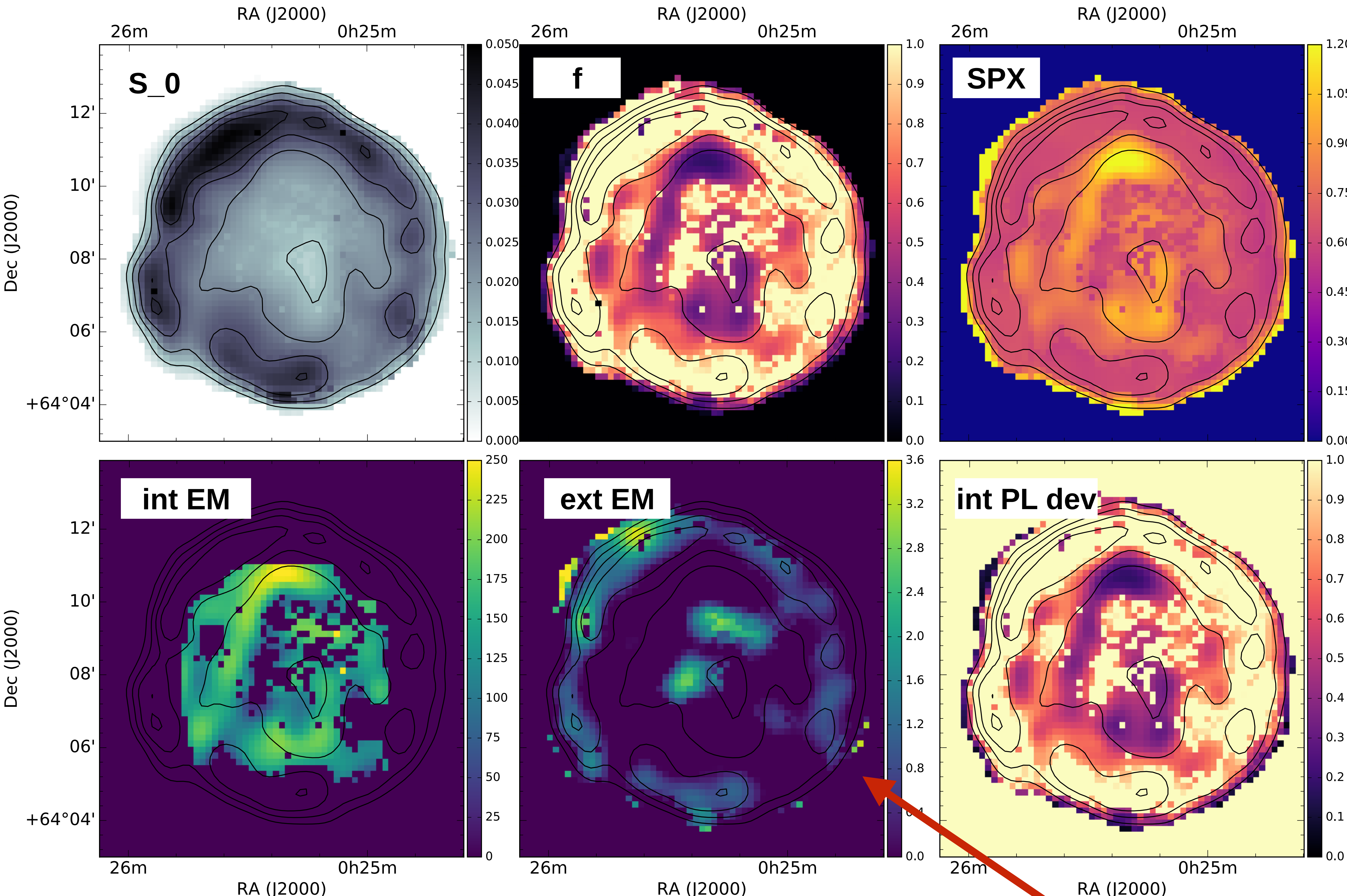

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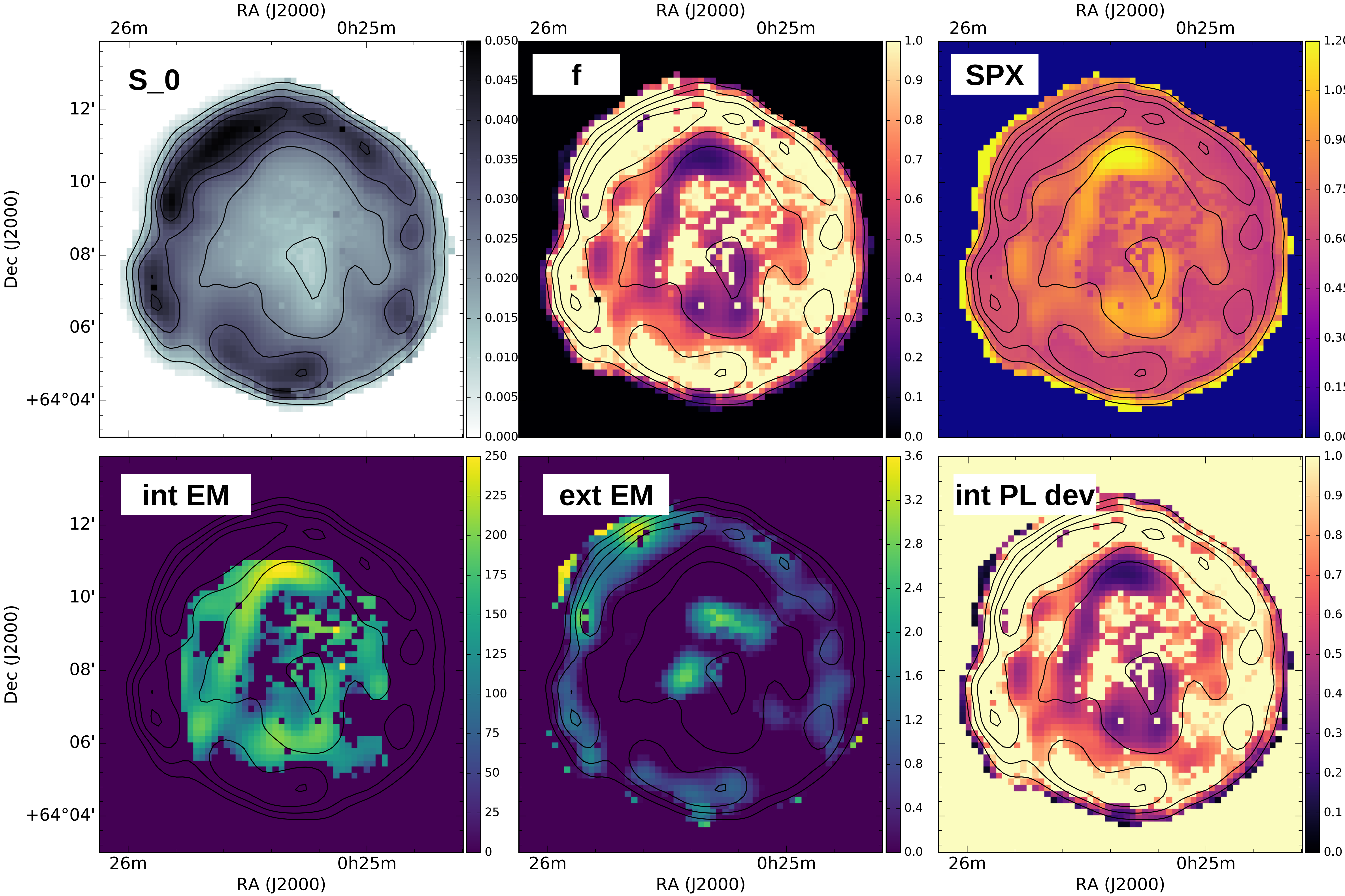

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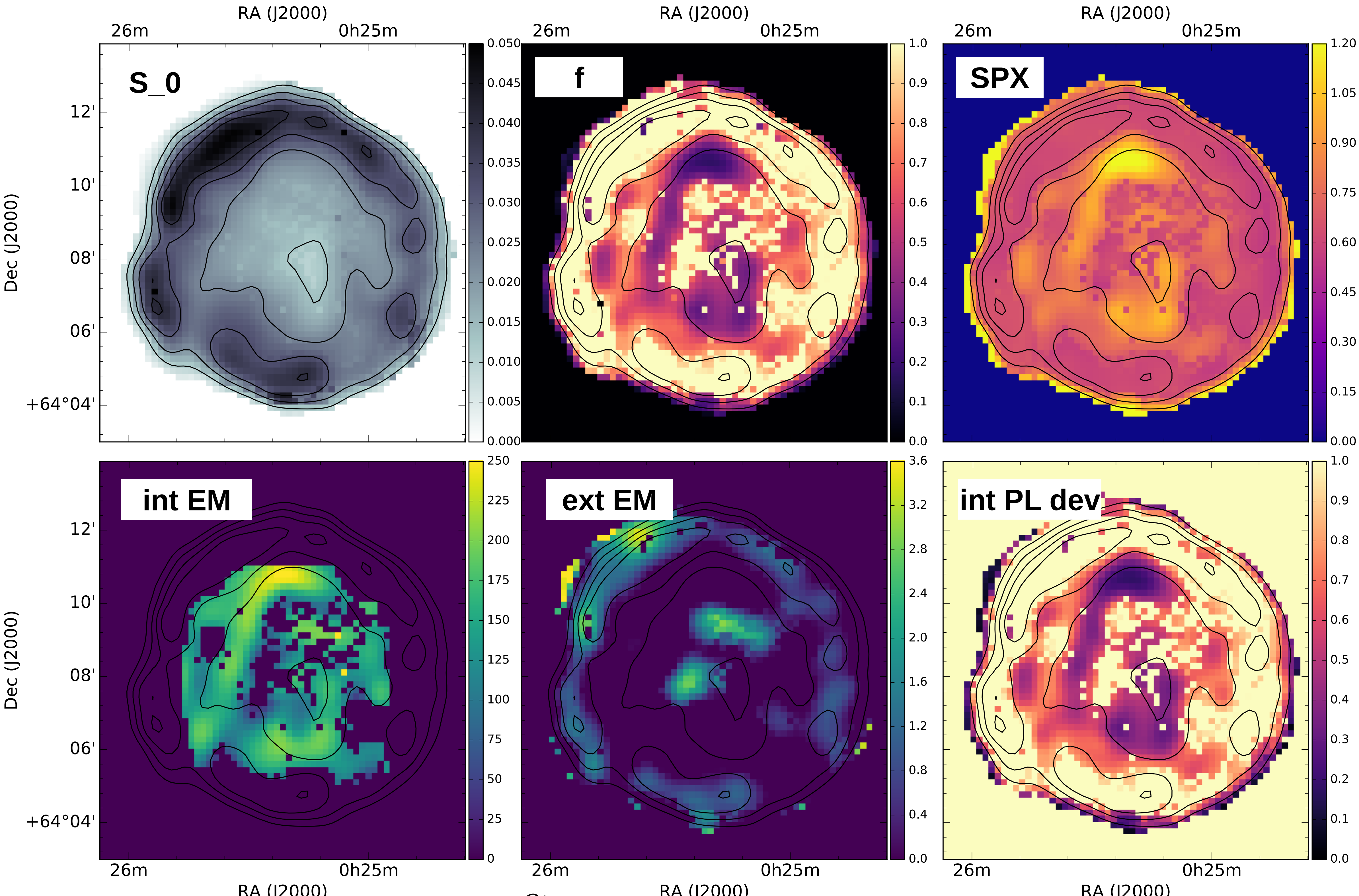

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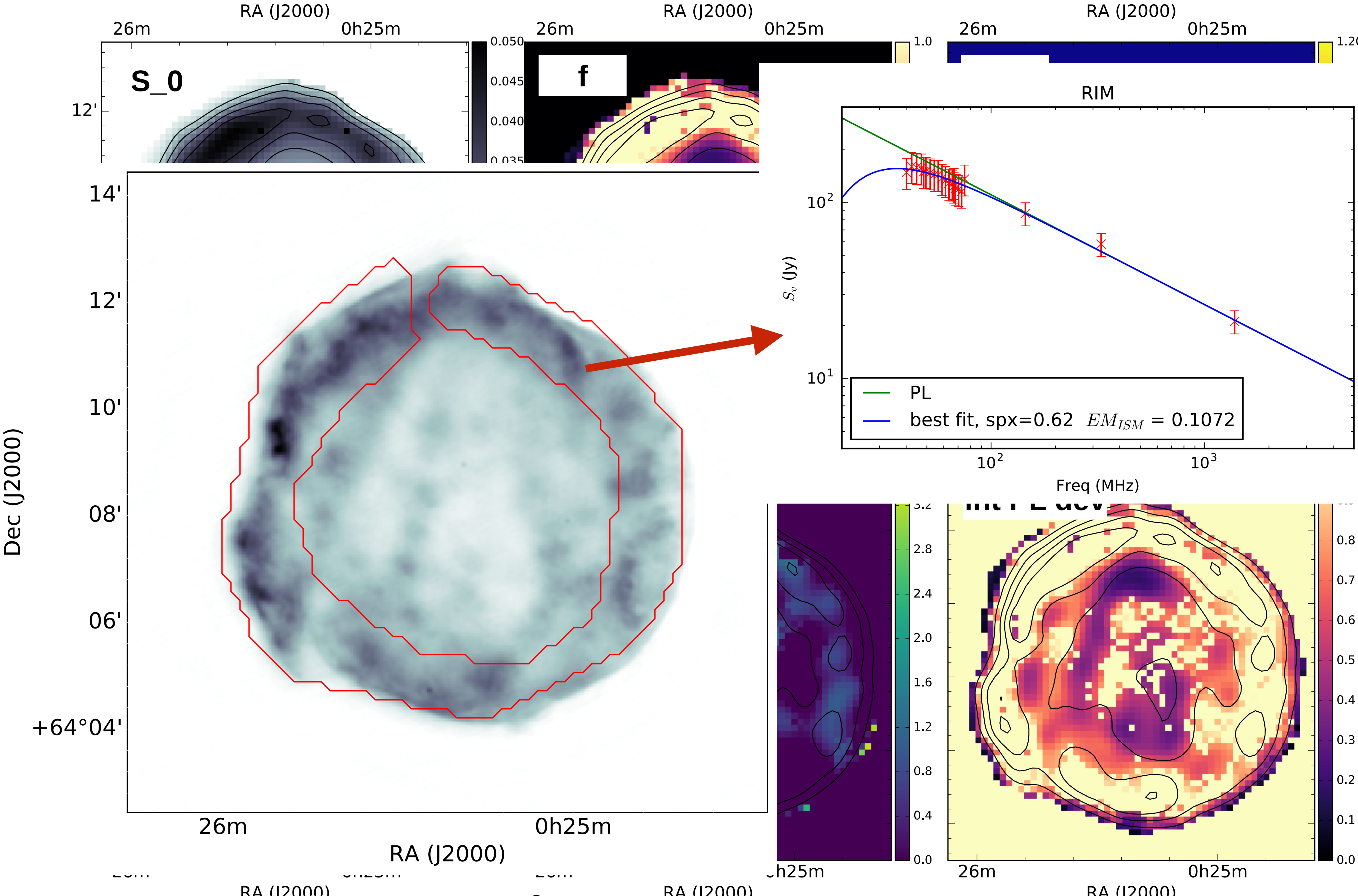

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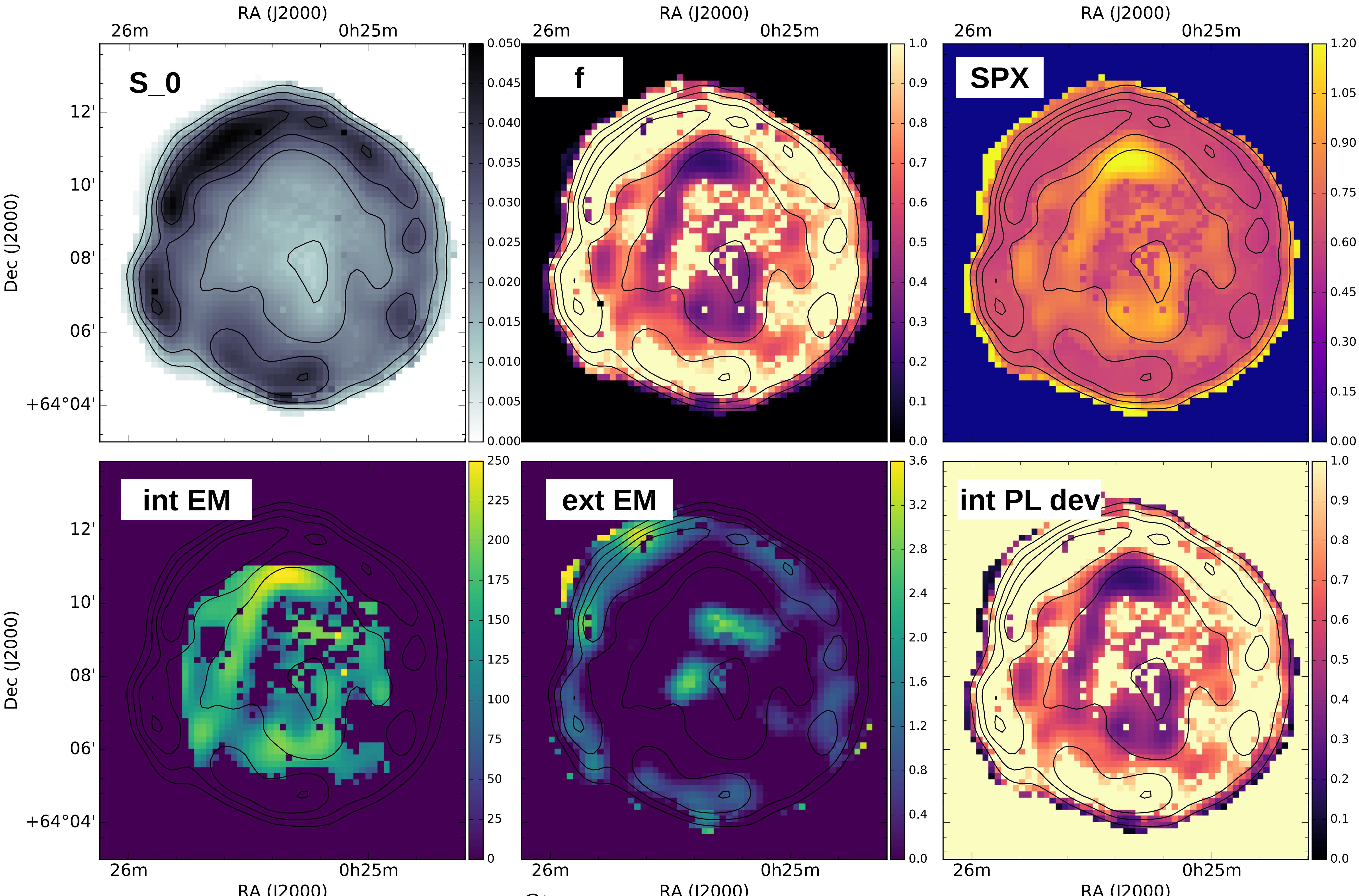

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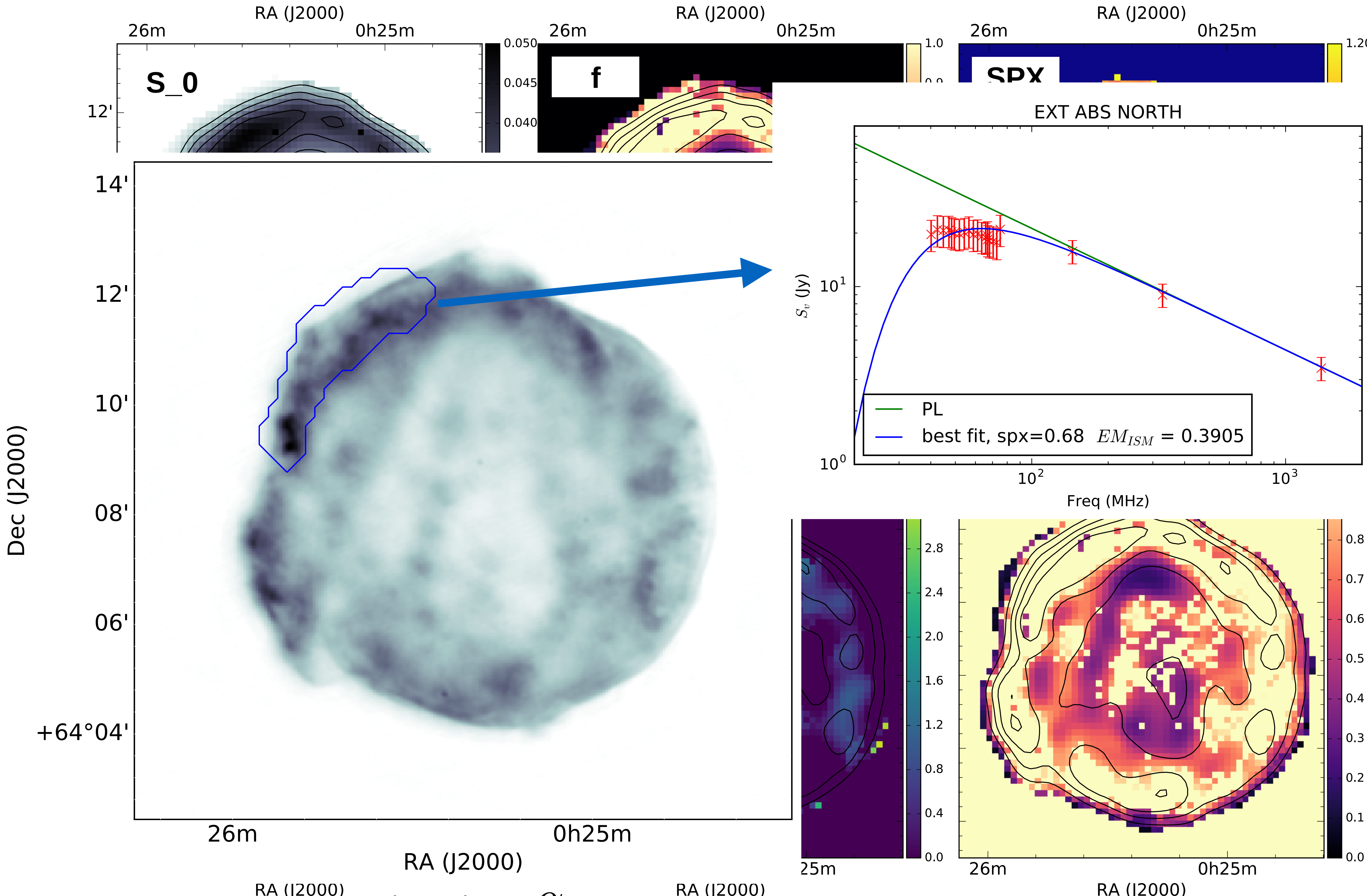

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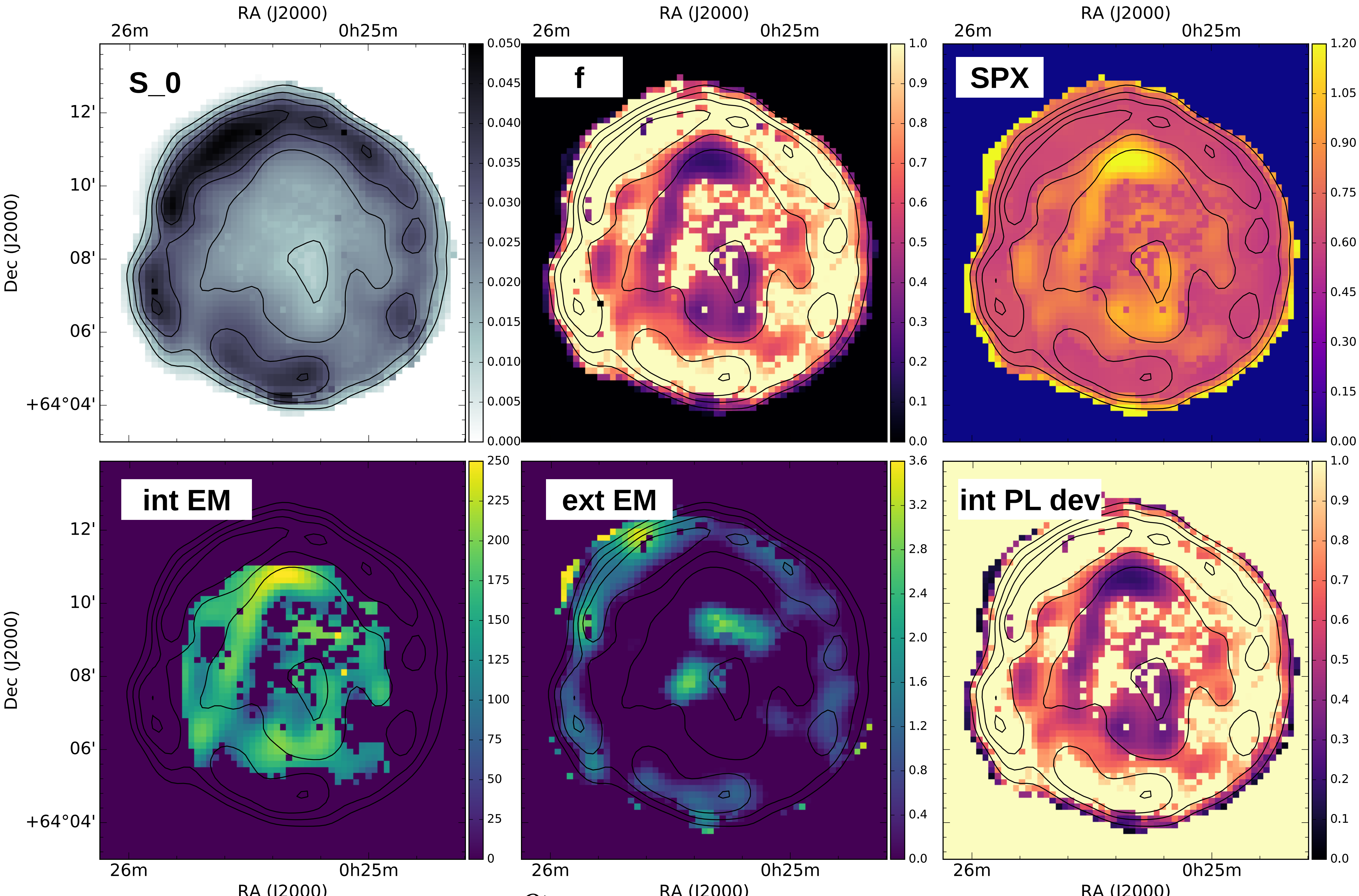

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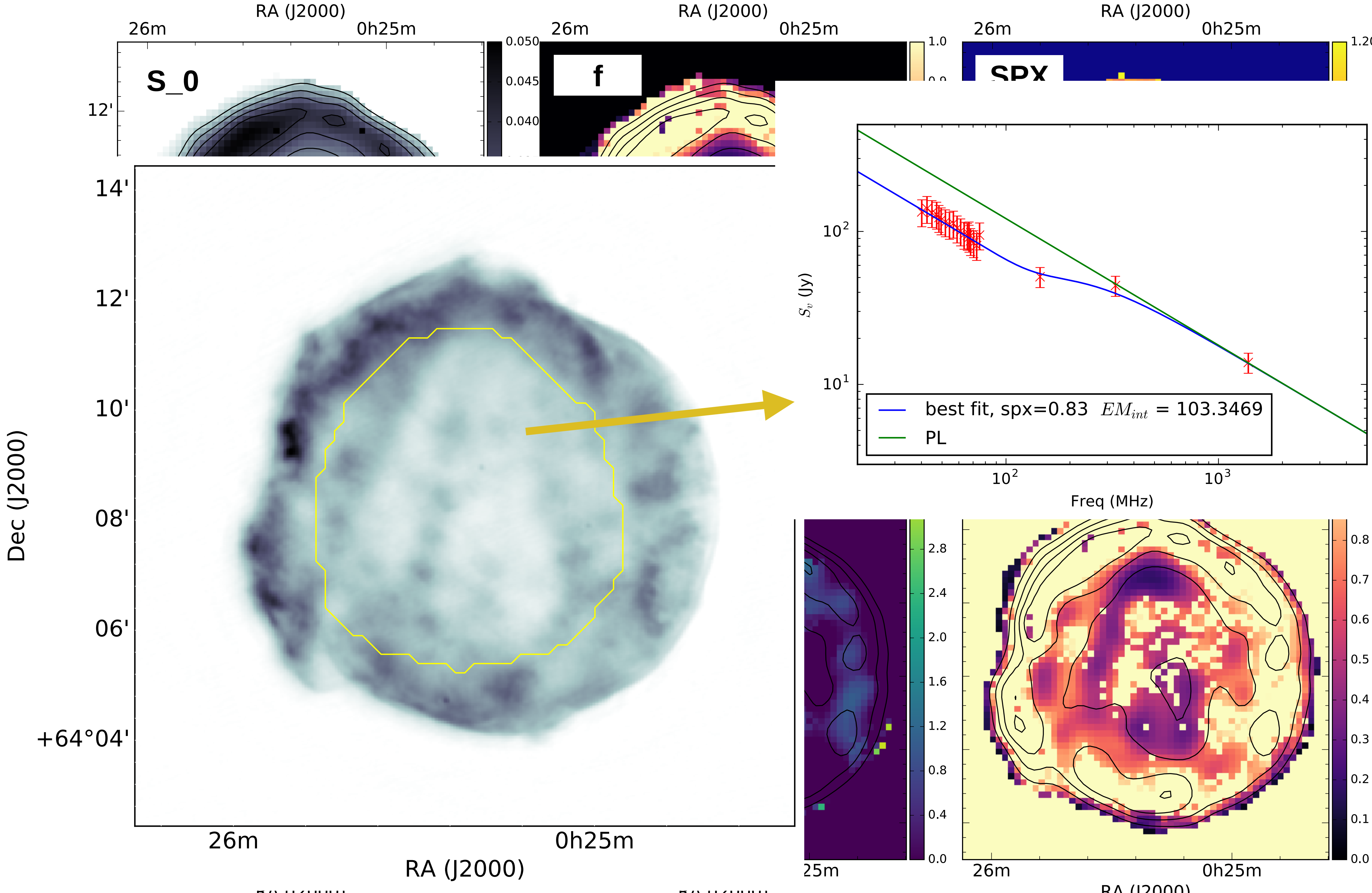

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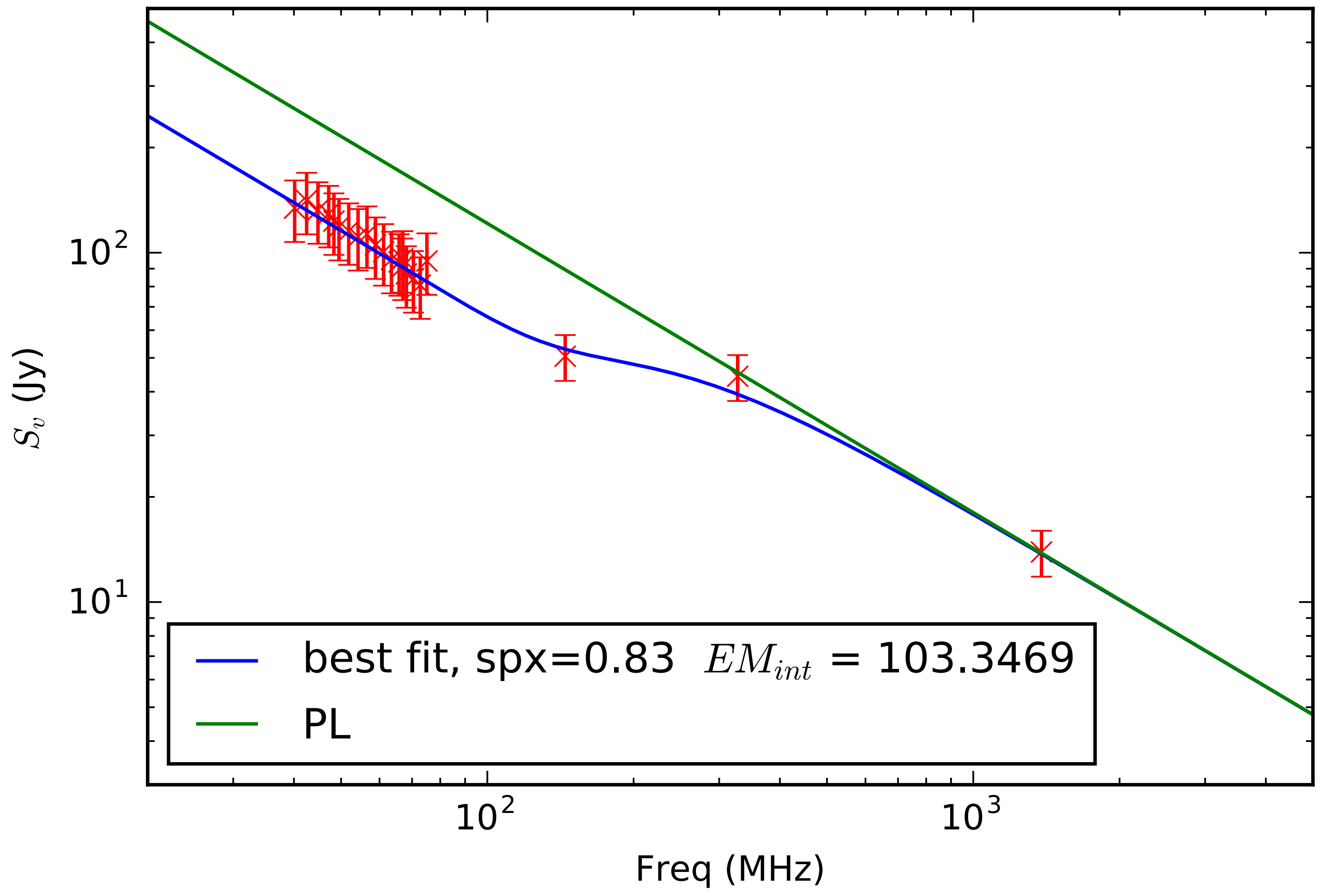
- To estimate mass from the measured absorption we need:
  - degree of ionisation and composition
  - temperature
  - geometry of the material
- Tycho was a Type Ia explosion: originally 1.4 M⊙



This plot corresponds to **EM=103 pc cm^{-6}** in the region within the reverse shock, which is very high

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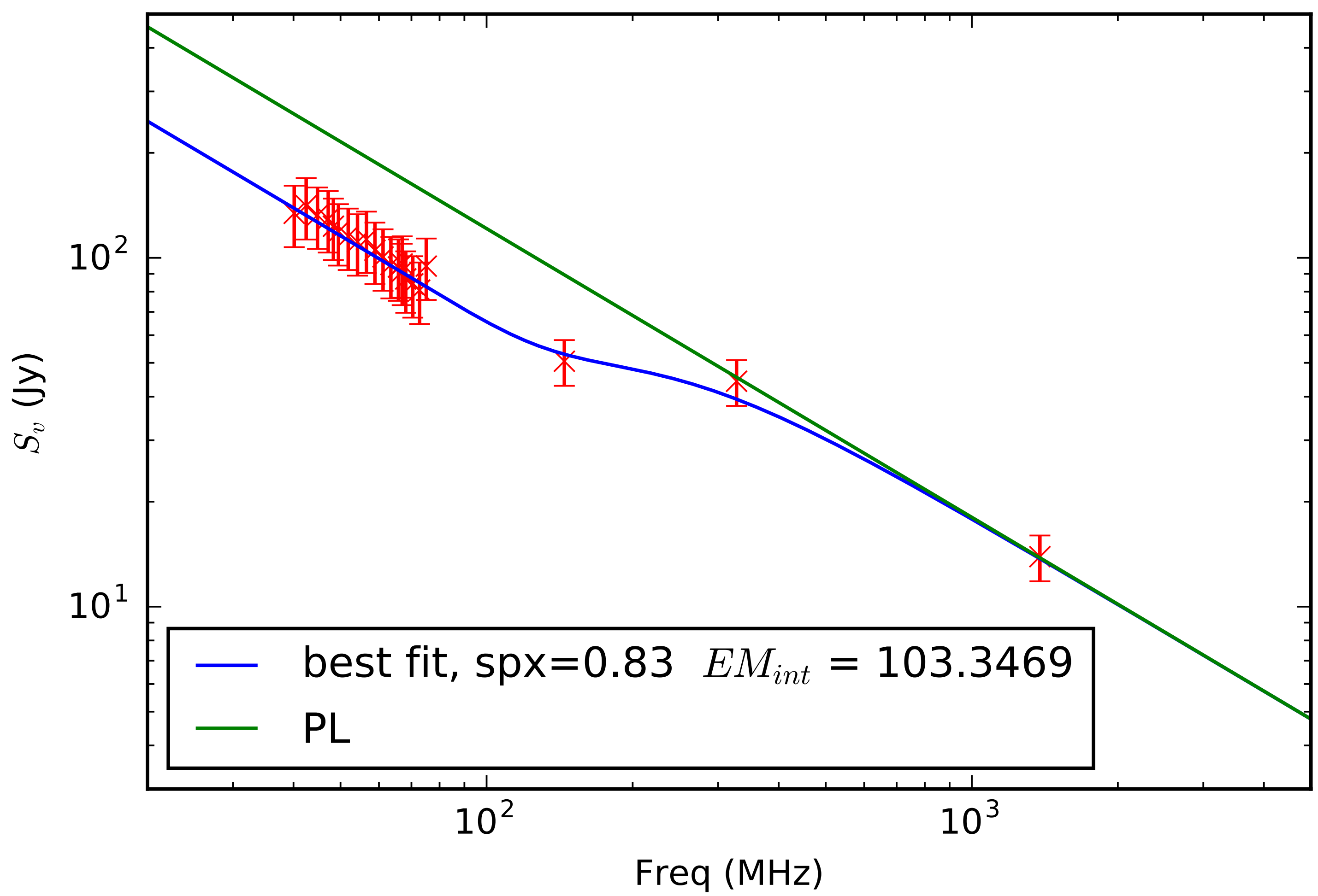

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    100 K — 91 M⊙

    10 K — 25 M⊙
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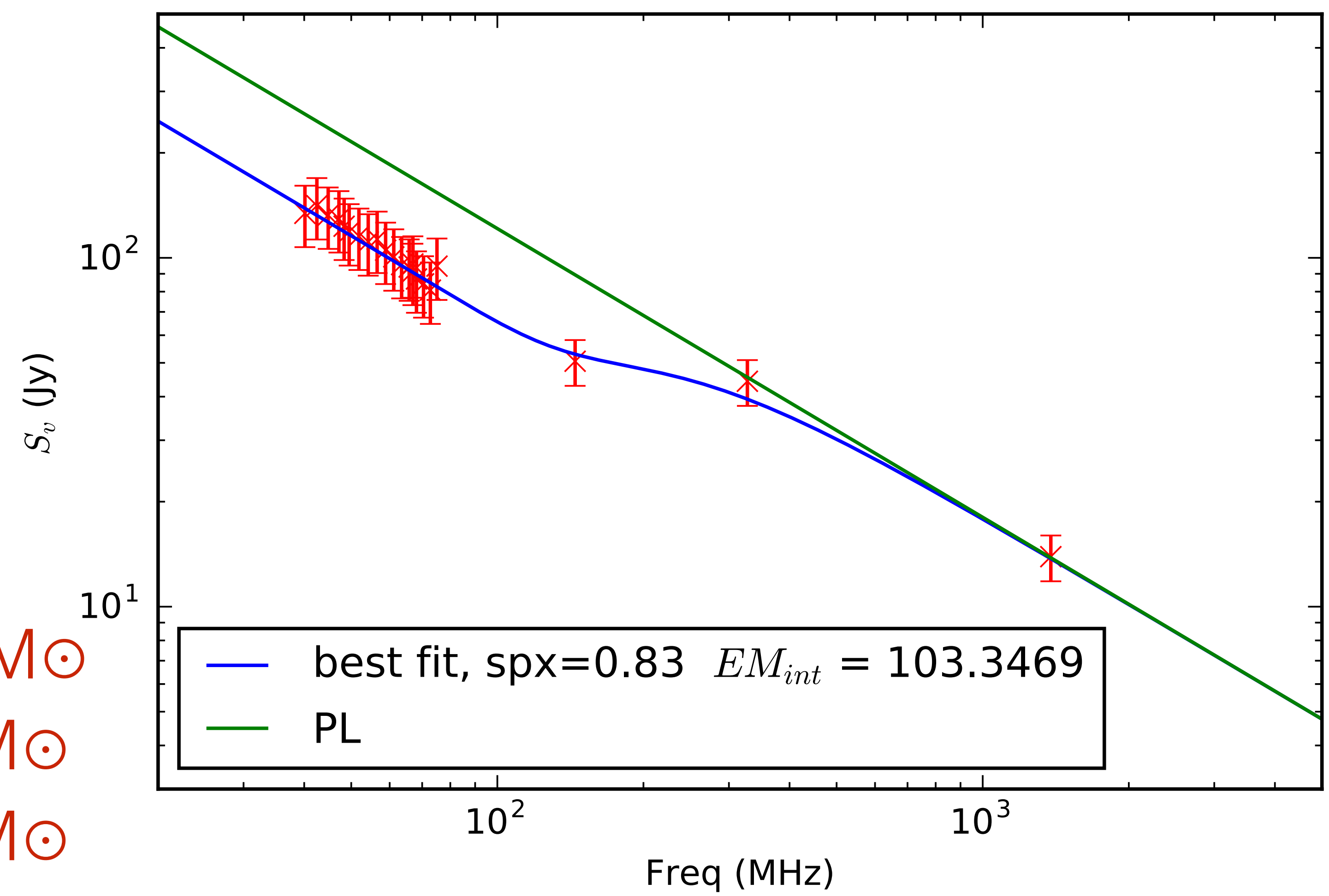

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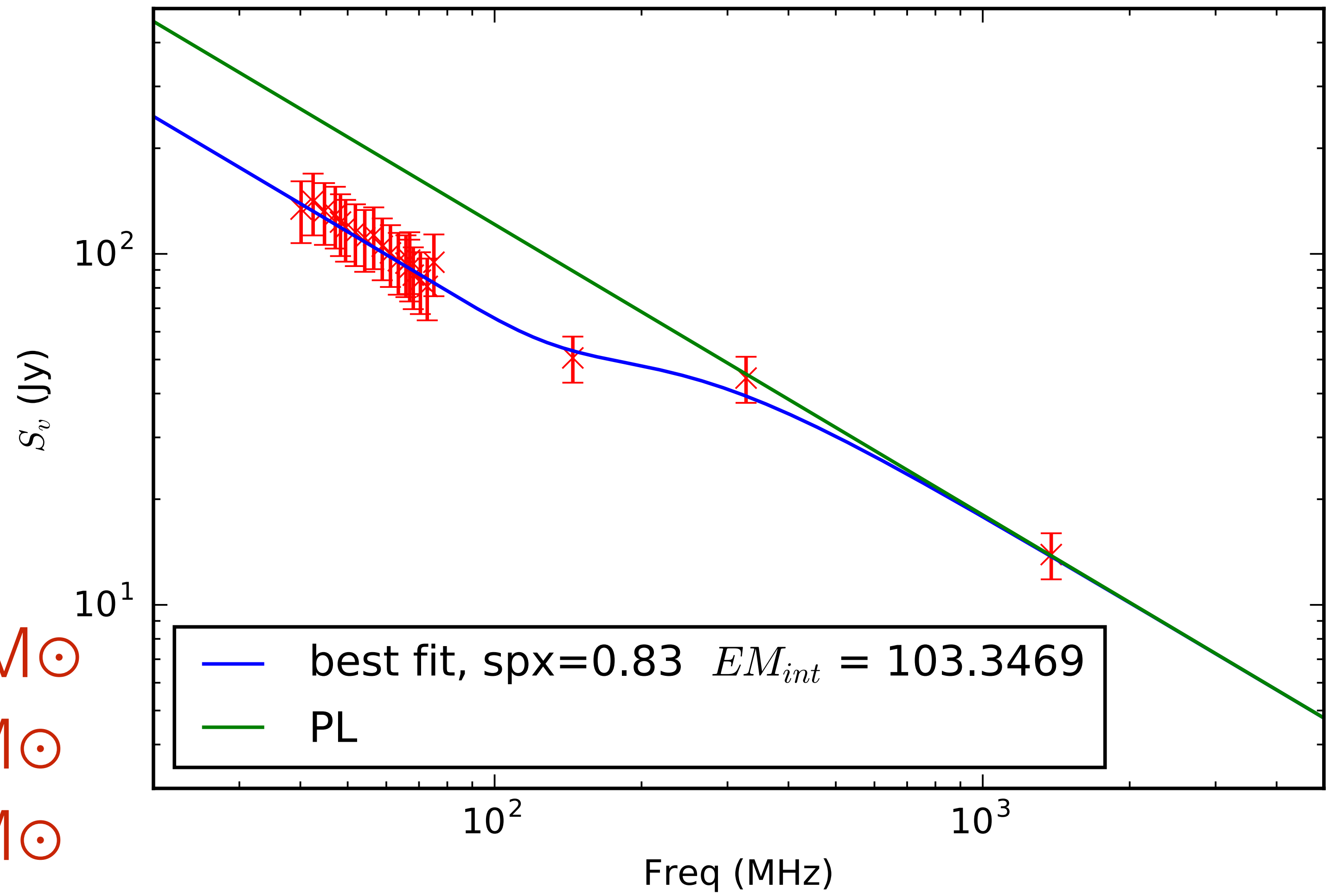

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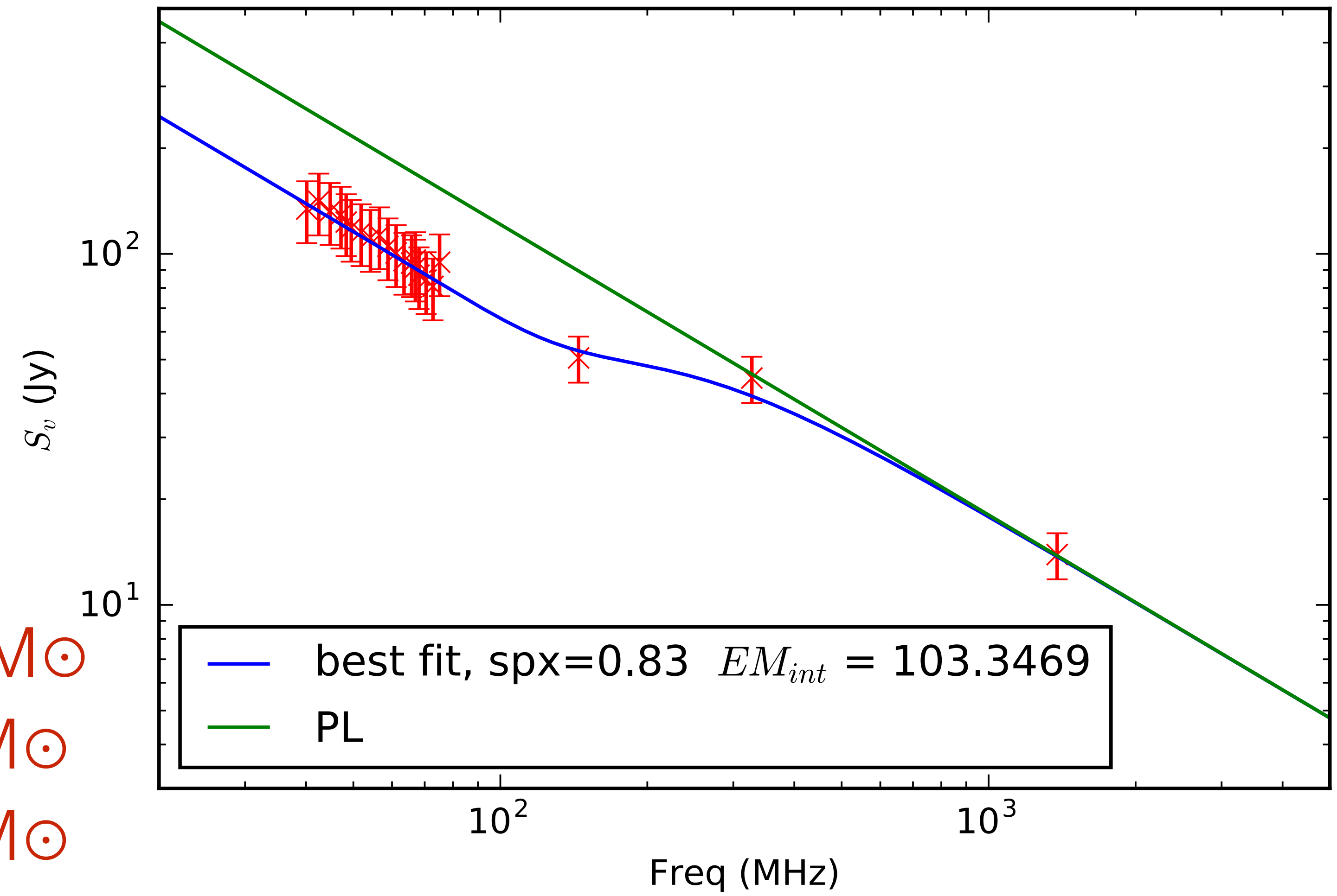

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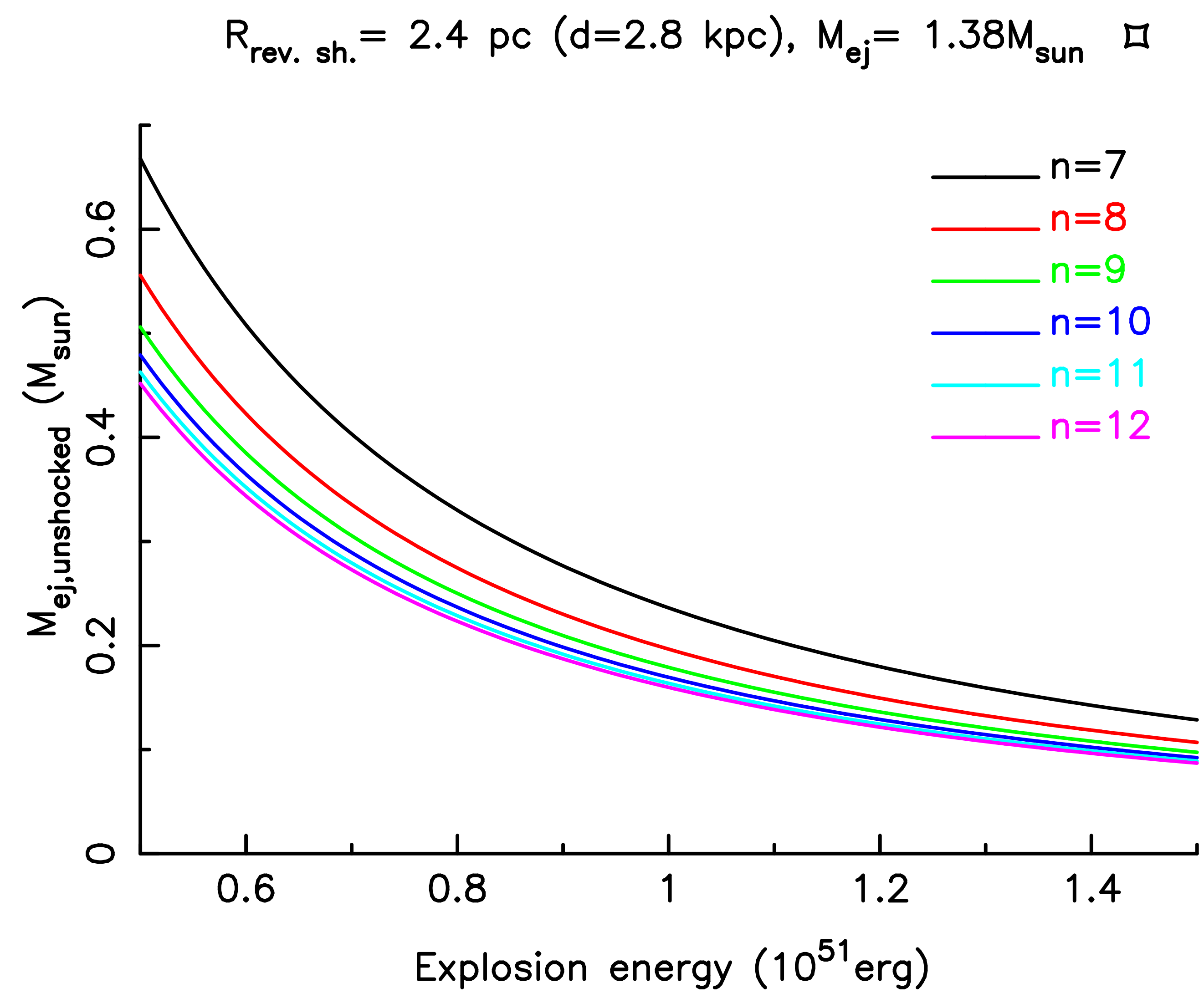
PRELIMINARY

Is it possible that ejecta have a very different structure? [Fe-rich knot, Yamaguchi+ 17] Foamy?

Can Fe be very highly ionised?

Might Tycho be ionising the medium around it?



Mass in unshocked ejecta from observed FS, RS, for an ejecta density profile with a PL distribution and a flat distribution in the core


Thanks!

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$$M = A S l^{1/2} m_{\mathrm{p}} \frac{1}{Z} \sqrt{EM}$$